JCARME
ISSN: 2228-7922
Journal of Computational and Applied Research in Mechanical Engineering
Vol. x, No. x, xxxx
jcarme.sru.ac.ir

Entropy Generation Analysis of Non-Newtonian Fluid in Rotational Flow

Abbas Kosarineia[a,*]

[a]Department of Mechanical Engineering, Ahvaz Branch, Islamic Azad University, Ahvaz, Iran

Article info:

Received: 00/00/2000
Accepted: 00/00/2018
Online: 00/00/2018

Keywords:
Rotational flow,
Entropy generation,
Deborah number,
Nonlinear equations,
Perturbation method.

Abstract

The entropy generation analysis of non-Newtonian fluid in rotational flow between two concentric cylinders is examined when the outer cylinder is fixed and the inner cylinder is revolved with a constant angular speed. The viscosity of non-Newtonian fluid is considered at the same time interdependent on temperature and shear rate. The Nahme law and Carreau equation are used to modeling dependence of viscosity on temperature and shear rate, respectively. The viscous dissipation term is adding elaboration to the formerly highly associate set of governing motion and energy equations. The perturbation method has been applied for the highly nonlinear governing equations of base flow and found an approximate solution for narrowed gap limit. The effect of characteristic parameter such as Brinkman number and Deborah number on the entropy generation analysis is investigated. The overall entropy generation number decays in the radial direction from rotating inner cylinder to stationary outer cylinder. The results show that overall rate of entropy generation enhances within flow domain as increasing in Brinkman number. It, however, declines with enhancing Deborah number. The reason for this is very clear, the pseudo plastic fluid between concentric cylinders is heated as Brinkman number increases due to frictional dissipation and it is cooled as Deborah number increases which is due to the elasticity behavior of the fluid. Therefore, to minimize entropy need to be controlled Brinkman number and Deborah number.

Nomenclature

Br	Brinkman number as frictional dissipation parameter of flow ($Br = \mu_0(\Omega R_1)^2/k_c\Delta T_0$)
C_n	Constants of integration, *n=0, 1, 2*
C_p	Heat capacity
D	Interval across outer and inner cylinders
De	Elasticity force to viscous force ratio presented as Deborah number ($De=\lambda R_i\Omega/D$)
h	Convection heat transfer coefficient
k_c	Conductivity heat transfer coefficient
n	Exponent of power law
Na	Nahme number ($Na = \beta\, Br$)
Nu	Nusselt number
P	Pressure
Pe	Peclet number ($Pe = \rho\, C_p \Omega R_1 D/k_c$)
r	Radial coordinate
R	Radius of cylinder

*Corresponding author
Email address: Kosarineia@gmail.com

Re	Main Reynolds number ($Re = R_i \Omega D/\nu_0$)
t	Time
T	Temperature
Ta	Taylor number ($Ta = Re^2 \varepsilon$)
u	Dimensionless velocity vector
U	Velocity vector
z	Axial distance

Greek symbols

$\dot{\Gamma}$	Rate-of-strain tensor ($\dot{\Gamma} = \nabla U + (\nabla U)^T$)
Θ	Dimensionless temperature
Φ	Thermal dissipation ($\Phi = \tau : \nabla U$)
Ω	Angular velocity of inner cylinder
β	Viscosity sensitivity to temperature
ε	Ratio of interval space-to-radius ($\varepsilon = D/R_1$)
η	Dimensionless viscosity
θ	Tangential coordinate
λ	Fluid elasticity (relaxation time)
μ	Viscosity
μ_0	Zero-shear-rate viscosity
μ_∞	Infinite-shear-rate viscosity
ρ	Density
τ	Shear stress ($\tau = \mu \dot{\Gamma}$)

Subscripts

,t	Specify partial differentiation of time
1	Specify interior cylinder
2	Specify exterior cylinder
r	Specify radial coordinate
q	Specify isoflux for thermal boundary condition
T	Specify isothermal for thermal boundary condition
z	Specify axial coordinate
θ	Specify circumferential coordinate

1. Introduction

The Circular Couette flow consists of a viscous fluid bounded in the narrow gap between two revolving cylinders. Circular Couette flow has wide applications ranging from desalination to magneto-hydrodynamics and also in viscosimetric analysis. Different flow regimes have been categorized over the years including twisted Taylor vortices, wavy outflow boundaries, etc. It has been a well-researched and documented flow in fluid dynamics [1]. The detailed investigation and analysis of Circular Couette flow has also been a charming subject in the non-linear fluids field. For sample, the influence of an axial flow on the stability of the flow between concentric cylinders is explored for pseudoplastic fluids is mentioned in the literature [2].

The entropy is one of the most important characteristics in thermodynamics, due to closely associate with the thermodynamic irreversibility. In other hand, it is directly pertinened to the availability or exergy destruction in thermal systems. One of the modern thermal design techniques is reducing exergy destruction as means of enhancing thermodynamic performance, which is referred to as entropy generation minimization. For the heat exchanger interacts with the surrounding flow field, the entropy generation minimization allows the combined thermal resistance and pressure drop influence to be evaluated simultaneously.

Tasnim [3] investigated the hydro-magnetic effects on entropy generation inside a porous channel. In this study to present expressions for temperature and entropy generation number and irreversibility distribution ratio, the governing equations are simplified and solved analytically. Mahmud and Fraser [4] studied the forced convection and second law characteristics of fluid flow inside a channel made of two parallel plates. Carrington and Sun [5] presented an analytical phrase for the entropy generation to study the second law analysis in internal and external flows. The influence of entropy generation in boundary layer flows were investigated by Arpaci and Selamet [6]. They illustrated the entropy generation in forced convection heat transfer, is resulted from contribution both temperature gradient and fluid friction. Abu-Hijleh [7] presented numerical work to evaluate the entropy generation for different magnitude of the buoyancy parameter, Reynolds number and cylinder dimension in an air cross flow. Khalkhali et al. [8] developed a thermodynamic model of conventional cylindrical heat pipes based on the second law of thermodynamics and studied the entropy generation in a heat pipe system. Ashrafi [9] examined the heat transfer of viscoplastic fluids in the Circular Couette flow while viscous dissipation term considered through the energy equation. Hazbavi [10] presented a numerical work to investigate the applied magnetics force effects for a nonlinear viscoelastic fluid in the Circular Couette flow. Kosarineia et al. [11] investigated the effects of the applied magnetic field for magneto-micropolar fluid between inclined parallel porous plates. The highly nonlinear coupled governing equations are solved numerically by explicit Runge–Kutta and the velocity, microrotation, and temperature results are used to evaluate second law analysis.

Commonly, the zero-shear-rate viscosity of industrial fluids such as melts and polymeric materials are several magnitude orders higher than the water viscosity. The thermal conductivity of polymeric solutions is in order of 0.1 W/m.K; in fact, thermal conductivity of polymeric solutions is poor. The weak conduction of the frictional heat causes considerable temperature rise in a flowing industrial fluids [12]. Mostly, these enhancing temperature lead to declines exponentially the fluid viscosity. Typically, thermal dissipation is neglected in most cases of Non-Newtonian fluid researches, although temperature gradients lead to viscosity variations and it can considerably vary the corresponding isothermal flow, which have resulted to new instabilities forms. The thermal dissipation term of Newtonian fluids is always positive. It adopts positive or negative values with non-Newtonian liquids (viscoelastic material). In other hand, the viscous dissipation quantity of Newtonian fluids is always positive and therefore represents an irreversible mechanical depreciation into internal energy. The viscous dissipation quantity of viscoelastic fluids does not have to be positive, since some energy may be stored as elastic energy [13]. Reddy et al. [14] studied the heat and mass transfer in chemically reacting radiative Casson fluid flow over a slandering/flat stretching sheet in a slip flow regime with aligned magnetic field. Babu et al. [15] analysed the two-dimensional MHD flow across a slandering stretching sheet within the sight of variable viscosity and viscous dissipation. Also, Reddy et al. [16] studied the effects of viscous dissipation and nonlinear thermal radiation for Casson fluid flow embedded with magnetic nano-particles. Ramandevi et al. [17] investigated the MHD flow and heat transfer of two distinct non-Newtonian fluids (Casson and viscoelastic fluids) across a stretching sheet with the new heat flux theory namely Cattaneo-Christov.

Kumar et al [18] studied the heat transfer impact on MHD Ferro-Fluid Flow over a shrinking sheet and the governing equations are transmuted into coupled nonlinear ODE's with the assist of suitable similarity transformations then numerically solved by R.K. Fehlberg Technique. Also, Kumar et al [19] numerically investigated the MHD boundary layer flow which electrically conducting fluid past a cone and a wedge with Cattaneo-Christov heat flux. At first, the governing equations of flow are converted into ODE via proper self similarity transforms and the resultd equations are solved numerically by using Runge Kutta and Newton's methods. Kumar et al [20] studied the thermal transport of magneto hydrodynamic non-Newtonian fluid flow over a melting sheet in the presence of exponential heat source. The group of PDE is mutated as dimension free with the assistance of similarity transformations and the highly nonlinear coupled resulting equations are solved with the help of fourth-order Runge–Kutta based shooting technique.

However, the research is limited for the non-linear fluid flow due to the intricacy originated from both the geometry and nonlinearity of governing equations. On the other side, for a non-Newtonian fluid conforming the non-isothermal model in Circular Couette flow, no analytical heat transfer study was found. The viscosity of non-isothermal Carreau fluid is considered simultaneously interdependent on temperature and shear rate. The investigation of thermal dissipation effects on the flow is main motivation of the present work. As other novelty point, the perturbation method has been applied to highly nonlinear governing equations and finding a handy third order approximation solution. In a nutshell, the major aim of current work is to specify characteristics of entropy generation for non-Newtonian fluid conforming the non-isothermal model while thermal dissipation term is considering in energy equation and entropy generation is investigating in Circular Couette flow. The governing equations are simplified in the narrowed gap limit and analytical expressions are presented for dimensionless temperature and entropy generation number in both isothermal and isoflux cases.

2. Governing Equations

At first, the constitutive equations are presented and solution method is described. Suppose the incompressible Non-Newtonian fluid between two concentric cylinders with inner and outer radii R_1 and R_2 , respectively (Fig. 1).

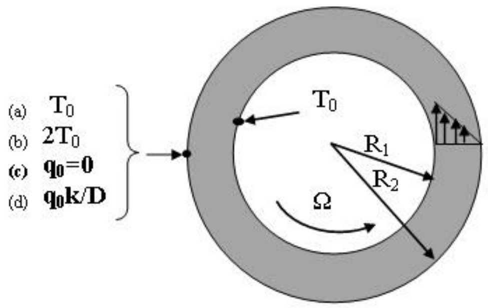

Fig. 1. Schematic of circular couette flow. (a) isothermal case with same wall temperature, (b) isothermal case with different wall temperature, (c) isoflux case with isolated outer cylinder and (d) isoflux case with applied constant heat flux to the outer cylinder.

The inner cylinder is rotated at a constant angular speed, Ω and the outer one is fixed. The constitutive equations for an incompressible Non-Newtonian fluid as follow [13]:

$$\boldsymbol{\nabla}.\boldsymbol{U} = \boldsymbol{0} \tag{1}$$

$$\boldsymbol{\rho}(\boldsymbol{U}_{,t} + \boldsymbol{U}.\boldsymbol{\nabla}\boldsymbol{U}) = -\boldsymbol{\nabla}\boldsymbol{P} + \boldsymbol{\nabla}.(\boldsymbol{\mu}\dot{\boldsymbol{\Gamma}}) \tag{2}$$

$$\rho C_p(T_{,t} + U.\nabla T) = k_c\nabla^2 T + \Phi \tag{3}$$

where a comma symbol gives the partial differentiation meaning and all other parameters and variable defined in Nomenclature section. Introducing dimensionless coordinates as follows:

$$r = \frac{R - R_1}{D}, z = \frac{Z}{D}, \eta = \frac{\mu}{\mu_0},$$
$$\bar{t} = \frac{\nu_0}{D^2}t, p = \frac{D^2}{\rho\nu_0^2}P, \Theta = \frac{T - T_0}{\Delta T_0}, \tag{4}$$
$$\dot{\gamma} = \frac{R_1\Omega}{D}\dot{\Gamma}, \quad u_r = \frac{D}{\nu_0}U_R,$$
$$u_\theta = \frac{1}{R_1\Omega}U_\theta, \quad u_z = \frac{D}{\nu_0}U_Z,$$

Where $\Delta T_0=1/\beta$ is a characteristic of temperature difference [13, pp. 207]. In the current work, the viscosity of fluid is considered simultaneously interdependent on temperature and shear rate. The Nahme law [13] and Carreau equation [13] are used to modelling dependence of viscosity on temperature and shear rate, respectively, as:

$$\eta(\dot{\gamma}, \Theta) = e^{-\beta\Theta}[1 + De^2 II_{2D}]^{(n-1)/2} \tag{5}$$

all parameters defined in Nomenclature section. The significant preference of above model respect to other Non-Newtonian equations is recovered viscosity Newton's law in the zero shear rates limit.
In current paper, it is supposed that $Ta=O(1)$, due to the narrowed gap limit simplification. Therefore, this assumption lead to the terms of $O(\varepsilon/Ta)$ in governing equations can be neglected and dimensionless governing equations are reduced as follows:

$$u_{r,r} + u_{z,z} = 0 \tag{6a}$$

$$u_{r,t} + u_r\, u_{r,r} + u_z\, u_{r,z} - Ta\,(u_\theta)^2 = -p_{,r} + \eta(u_{r,rr} + u_{r,zz}) + 2\eta_{,r}u_{r,r} + \eta_{,z}(u_{r,z} + u_{z,r}) \tag{6b}$$

$$u_{\theta,t} + u_r\, u_{\theta,r} + u_z\, u_{\theta,z} = u_r + \eta(u_{\theta,rr} + u_{\theta,zz}) + \eta_{,r}u_{\theta,r} + \eta_{,z}u_{\theta,z} \tag{6c}$$

$$u_{z,t} + u_r\, u_{z,r} + u_z\, u_{z,z} = -p_{,z} + \eta\left(u_{z,rr} + u_{z,zz}\right) + \eta_{,r}\left(u_{r,z} + u_{z,r}\right) + 2\eta_{,z}u_{z,z} \qquad (6d)$$

$$Pe\left(\Theta_{,t} + u_r\, \Theta_{,r} + u_z\, \Theta_{z}\right) = \left(\Theta_{,rr} + \Theta_{,zz}\right) + Na\left(u_{\theta,r} + u_{\theta,r}\right)\eta \qquad (6e)$$

In accordance with principle of steady tangential annular flow [13], the base flow velocity and temperature functions for steady state can be expressed as:

$$u_r = 0\,,\ u_\theta = f(r),\ \ u_z = 0\,,\ \Theta = g(r) \qquad (7)$$

The dimensionless governing equations after introducing the above simplifications (7) into (6), gives:

$$\frac{dp}{dr} = Ta\,(u_\theta)^2 \qquad (8a)$$

$$\frac{d}{dr}\left(e^{-\beta\Theta}\left[1 + De^2\left(\frac{du_\theta}{dr}\right)^2\right]^{(n-1)/2}\frac{du_\theta}{dr}\right) = 0 \qquad (8b)$$

$$\frac{d^2\Theta}{dr^2} = -Na\, e^{-\beta\Theta}\left[1 + De^2\left(\frac{du_\theta}{dr}\right)^2\right]^{(n-1)/2}\left(\frac{du_\theta}{dr}\right)^2 \qquad (8c)$$

Two common surface thermal conditions are used in the analysis of current problem, both specified temperature and specified heat flux for the outer cylinder are used with same hydrodynamic boundary conditions on the velocity field.

3. Solution Procedure

First, simplify the problem in which viscosity, as well as the thermal conductivity, is not varying with temperature. The problem can be written in dimensionless form as:

$$\frac{d}{dr}\left(\left[1 + De^2\left(\frac{du_\theta}{dr}\right)^2\right]^{(n-1)/2}\frac{du_\theta}{dr}\right) = 0 \qquad (9a)$$

$$\frac{d^2\Theta}{dr^2} = -Br\left[1 + De^2\left(\frac{du_\theta}{dr}\right)^2\right]^{(n-1)/2}\left(\frac{du_\theta}{dr}\right)^2 \qquad (9b)$$

subject to the boundary conditions:

$$at \quad r = 0 \quad u_\theta = 1\,, \quad \Theta = 0 \qquad (10a)$$

$$at \quad r = 1 \quad u_\theta = 0\,, \quad \Theta = 0 \qquad (10b)$$

Equations (9) through (10) are easily solved to give:

$$u_\theta = 1 - r \qquad (11a)$$

$$\Theta = \frac{Br}{2}\left[1 + De^2\right]^{(n-1)/2}(r - r^2) \qquad (11b)$$

The maximum temperature occurs at $r=0.5$ and is given by:

$$\Theta_{max} = \frac{Br}{8}\left[1 + De^2\right]^{(n-1)/2} \qquad (12)$$

The simple result can be used for making rough estimates of the temperature rise that can be expected in the gap between two moving surface in the absence of an axial pressure gradient. The equations (8) are perturbed form of equations (9) which are become as:

$$\left[1 + De^2\left(\frac{du_\theta}{dr}\right)^2\right]^{(n-1)/2}\frac{du_\theta}{dr} = ce^{\beta\Theta} = c\left(1 + \beta\Theta + \frac{1}{2}\beta^2\Theta^2 + \frac{1}{6}\beta^3\Theta^3 + \cdots\right) = C \qquad (13a)$$

$$\frac{d^2\Theta}{dr^2} = -Na\, e^{-\beta\Theta}\left[1 + De^2\left(\frac{du_\theta}{dr}\right)^2\right]^{(n-1)/2}\left(\frac{du_\theta}{dr}\right)^2 \qquad (13b)$$

where c is integration constant. Thus for small values of β the aforementioned solution of velocity is expected to be accurate. This can be inserted into the energy equation (13b) and it is also necessary to expand the integration constant in a similar series to give:

$$\frac{d^2\Theta}{dr^2} = -\beta\, Br\, C = -Br\,(\beta\, C_0 + \beta^2 C_1 + \cdots) = -Na\,(C_0 + \beta\, C_1 + \beta^2 C_2 + \cdots) \quad (14)$$

Equations (13) are solved by a perturbation procedure, using the temperature sensitivity of the viscosity, β, as the perturbation parameter. This considered the form of the expansion solution to the following:

$$\Theta = \Theta_0(r) + \beta\,\Theta_1(r) + \beta^2\Theta_2(r) + \cdots \quad (15)$$

When this expansion is substituted into equation (14), sets of differential equations are obtained by equating coefficients of equal powers of perturbation parameter. The resulting differential equations are solved with the related boundary conditions. The handy third order approximation solution to the steady tangential annular flow governed by equations (8) is obtained and resulted to the velocity and temperature expressions in dimensionless form as:

$$u_\theta = 1 - r \quad (16)$$

$$\Theta_{T1} = \frac{Na\,C_0}{2}(r - r^2) - \frac{Na^2 {C_0}^2\beta}{24}(r - 2r^3 + r^4) \quad (17a)$$

$$\Theta_{T2} = b\,r - \frac{Na\,C_0}{2}(r - r^2) + \frac{Na\,C_0\beta^2}{6}(r^3 - r) - \frac{Na\,C_0\beta^4}{24}(r^4 - r) - \frac{Na^2 {C_0}^2\beta}{24}(r - 2r^3 + r^4) \quad (17b)$$

$$\Theta_{q1} = \frac{Na\,C_0}{2}(2r - r^2) - \frac{Na^2 {C_0}^2\beta}{24}(8r - 4r^3 + r^4) \quad (17c)$$

$$\Theta_{q2} = r + \frac{Na\,C_0}{2}(2r - r^2) + \frac{Na\,C_0\beta}{6}(r^3 - 3r) - \frac{Na\,C_0\beta^2}{24}(r^4 - 4r) - \frac{Na^2 {C_0}^2\beta}{24}(8r - 4r^3 + r^4) \quad (17d)$$

Here C_0 is equal to $[1 + De^2]^{\frac{n-1}{2}}$. The equation (17a), assigned as Θ_{T1} which is the dimensionless temperature for the isothermal case with same thermal boundary conditions, $\Theta=0$ at *inner cylinder* and *outer cylinder*. The equation (17b), assigned as Θ_{T2} which is the dimensionless temperature for the isothermal case with different thermal boundary conditions, $\Theta=0$ at *inner cylinder* and $\Theta=\beta$ at *outer cylinder*. The equation (17c), assigned as Θ_{q1} which is the dimensionless temperature for the isoflux case with thermal boundary conditions, $\Theta=0$ at *inner cylinder* and $\Theta'=0$ at *outer cylinder* (the temperature at the inner cylinder is specified as isothermal case but the heat flux at outer cylinder is specified as zero value). The equation (17d), assigned as Θ_{q2} which is the dimensionless temperature for the isoflux case with thermal boundary conditions, $\Theta=0$ at *inner cylinder* and $\Theta'=\beta$ at *outer cylinder* (the temperature at the inner cylinder is specified as isothermal case but the heat flux at outer cylinder is specified as non-zero value).

Definition of dimensional local entropy generation rate, S_{gen} (W m^{-3}K^{-1}), is expressed as [21]:

$$S_{gen} = \frac{k_c}{T_0}(\nabla T)^2 + \frac{\Phi}{T_0} \quad (18)$$

The dimensional local volumetric entropy generation rate, S_{gen}, for the steady tangential annular flow in cylindrical coordinates is expressed as:

$$S_{gen} = \frac{k_c}{T_0}\left(\frac{dT}{dR}\right)^2 + \frac{\tau_{R\theta}}{T_0} R \frac{d}{dR}\left(\frac{U_\theta}{R}\right) \quad (19)$$

Equation (19) indicated two different sources of heat transfer contributed in entropy generation. The first term of equation (19) is expressed the heat transfer entropy generation and the second term of equation (19) is expressed the friction dissipation entropy generation. The dimensional entropy generation rate S_{gen}, can be written in dimensionless form as:

$$N = \left(\frac{d\Theta}{dr}\right)^2 + Br\left[1 + De^2\left(\frac{du_\theta}{dr}\right)^2\right]^{(n-1)/2}\left(\frac{du_\theta}{dr}\right)^2 \quad (20)$$

Finally, dimensional form is achieved by substituting Eq. (9b) into Eq. (20):

$$N = \left(\frac{d\Theta}{dr}\right)^2 - \frac{d^2\Theta}{dr^2} \quad (21)$$

Against dimensional form, the Equation (21) indicated two different sources of heat transfer contributed in entropy generation with opposite tendencies. The first term of equation (21), specified as N_{SRHT} which is the entropy generation due to radial heat transfer, whereas the second term of equation (21), specified as N_{SDISS} which is the entropy generation due to friction dissipation. The entropy generation number (N) is achieved by substituting equations (17) into equation (21). For sample, dimensional form of the entropy generation number is achieved by substituting equation (17a) into equation (21), as:

$$N_{T1} = \frac{Na\,C_0}{576}\left(576 + 288\,Na\,C_0\beta(r^2 - r)\right) + \frac{Na^2 C_0{}^2}{576}\left(-12 + 24r + Na\,C_0\beta(1 - 6r^2 + 4r^3)\right)^2 \quad (22)$$

The equation (22), specified as N_{T1} which is expressed the entropy generation in the isothermal case with same thermal boundary conditions (the temperature at the inner and outer cylinder is specified as zero value).

In current problem, both fluid frictions and radial heat transfer with opposite tendencies contributed in entropy generation. The volumetric entropy generation rate of fluid is evaluated by the equation (19). However, this equation does not specify clear the entropy generation source i.e. the entropy generation due to radial heat transfer or entropy generation due to fluid friction dominates. The ratio of irreversibility distribution (Φ_{irr}) is defined as the relationship of entropy generation due to friction dissipation (N_{SDISS}) to entropy generation due to heat transfer (N_{SRHT}) [21]. Therefore, in special case with $\Phi_{irr}=1$, both entropy generation due to friction dissipation and entropy generation due to heat transfer equally contribute to total entropy generation and for $0 \le \Phi_{irr} < 1$, the entropy generation due to heat transfer dominates and when $\Phi_{irr} > 1$, the entropy generation due to fluid friction dominates. The contribution of heat transfer entropy generation (N_{SRHT}) to overall entropy generation rate (N_S) is required to optimization and engineering design of several industrial applications problems. Hence, the ratio of heat transfer entropy generation to the total entropy generation is defined as an alternative parameter for irreversibility distribution in dimensionless form as [21]:

$$Be = \frac{N_{SRHT}}{N_{SRHT} + N_{SDISS}} = \frac{1}{1 + \Phi_{irr}} \quad (23)$$

This parameter is called Bejan number which represented whether the entropy generation due to fluid friction or entropy generation due to heat transfer is dominated. Therefore, in case $Be=1$, corresponding to the special case at which overall entropy generation due to heat transfer only. In case $Be=0$, corresponding to the special case at which overall entropy generation due to friction dissipation only.

In the current work, evaluation is performed for a typical Carreau fluid with λ=0.0173, n=0.538 [13].

4. Results and Discussion

In this work, the perturbation method has been applied to highly nonlinear governing equations for finding an approximate solution. It was only considered the third-order approximation, by utilizing four terms in nonlinear part series expansion. The comparisons between the numerical solution with approximations (17a) are shown in Figs. 2 and 3. The maximum absolute error is shown in these figures about $3.5E\text{-}7$ at $r=0.5$. These comparisons are proved the proficiency of perturbation method.

The temperature profiles are depicted in Fig. 2, with different Brinkman number values for the isothermal boundary condition case while both walls are kept at the same specified temperature (T_0). The temperature profile (Fig. 2) display a maximum value within interval space between inner and outer cylinders. The temperature monotonically increases as the Brinkman number enhances due to thermal dissipation increases with enhance enhancing shear rate. This is exhibited to the reality that the thermal dissipation magnitude in Eq. (6c) enhances with increasing Brinkman number.

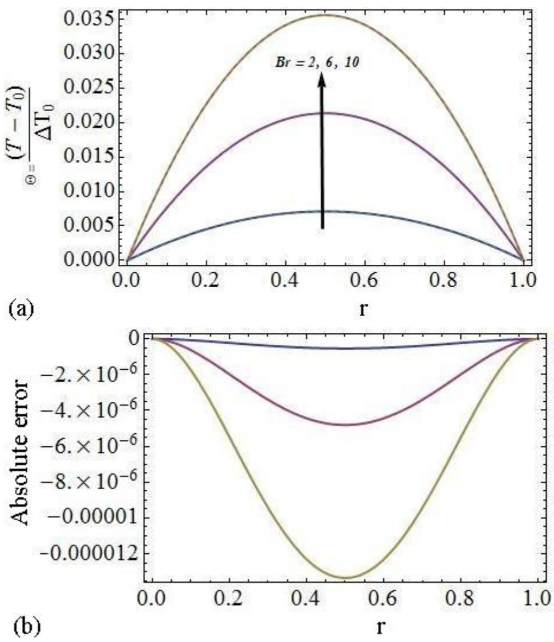

Fig. 2. Approximate solution and *RK4* numerical solution (a) and absolute error (b) of temperature profile for fluid with *n*=0.538, β=0.1, *De=0.5* for various Brinkman numbers.

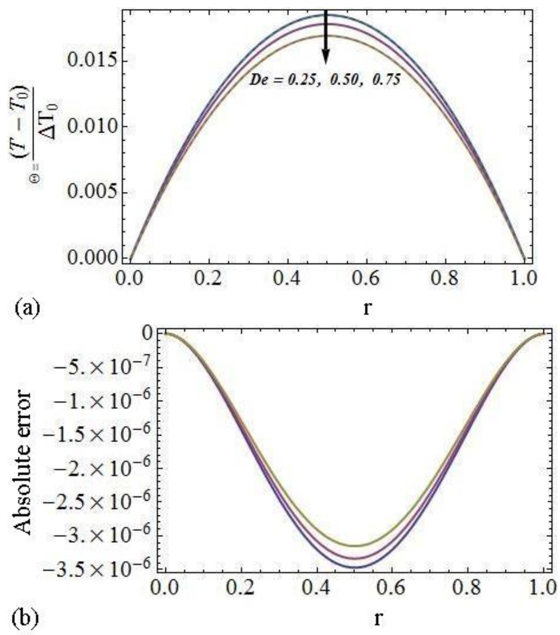

Fig. 3. Approximate solution and *RK4* numerical solution (a) and absolute error (b) of temperature profile for fluid with *n*=0.538, β=0.1, *Br=5* for various Deborah numbers.

The influence of Deborah number on temperature plot is depicted in Fig. 3, for the isothermal boundary condition case while both walls are kept at the same specified temperature (T_0), which shown the maximum temperature value declines within interval space between inner and outer cylinders with raising the Deborah number. This fluid behavior resulted from the elasticity effect of non-Newtonian fluid, where the fluid viscosity declines with raising elasticity of fluid. The viscosity becomes correspondingly lower, so that the solution above the center of the interval space between inner and outer cylinders is dragged along with the inner cylinder.

The influence of characteristic parameters such as Deborah number and Brinkman number on total entropy generation is depicted for isothermal case in Figs. 4 and 5, respectively.

The entropy generation decays in the radial direction from rotating inner cylinder to stationary outer cylinder as is shown in Fig. 4. It depicted mentioned trend decreases with increasing the Deborah number as elasticity parameter of fluid.

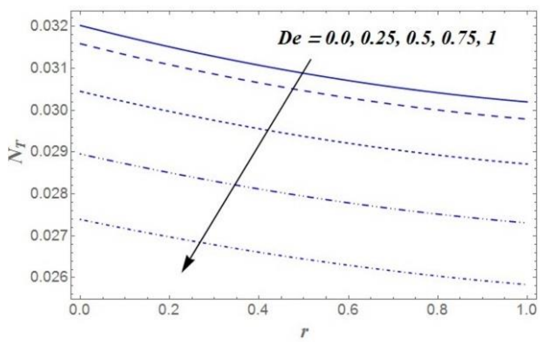

Fig. 4. Deborah number influence on overall entropy generation plot for fluid with *n*=0.538, β=0.1 and *Br=1* (isothermal case).

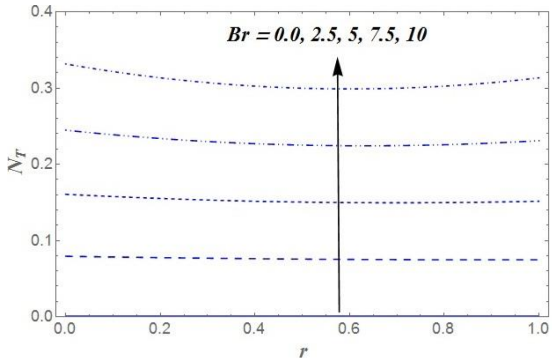

Fig. 5. Influence of Brinkman number on overall entropy generation plot for fluid with n=0.538, β=0.1 and $De=0.1$ (isothermal case).

As it can be seen from Fig. 5, the Brinkman number influence on total entropy generation profile is depicted which shown the total entropy generation plot climbs with Brinkman Number increasing. Although, the overall entropy generation number is slightly greater for domain of flow in adjacent of inner cylinder than that outer cylinder. The cause of this is very clear, Brinkman number is acted as a heat source and Brinkman Number increasing caused that the heat is generated within the moving fluid layers. Therefore, Brinkman number must be controlled to minimize total entropy generation.

Subsequent, the characteristic parameters influence such as Deborah number and Brinkman number on total entropy generation is depicted for Isoflux in Figs. 6 and 7, respectively. The entropy generation decays in the radial direction from rotating inner cylinder to stationary outer cylinder as is shown in Fig. 6. This efficacy is much more pronounced in Isoflux boundary conditions than that for the isothermal case. This behavior of overall entropy generation number resulted from the influence of Non-Newtonian fluid elasticity, where the fluid viscosity declines with raising elasticity of fluid. Keunings and Crochet [22] is reported the viscosity dependency and the effect of Non-Newtonian fluid elasticity, also, Pinho and Oliveira [23] are investigated these discussion in greater detail.

As it can be seen from Fig. 7, a like expression can be presented for the overall entropy generation trend in the radial direction from rotating inner cylinder to stationary outer cylinder, although the influence of Brinkman number is vice versa the elasticity effect.

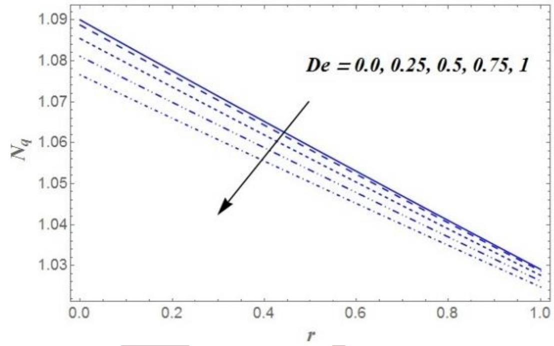

Fig. 6. Deborah number influence on overall entropy generation plot for fluid with n=0.538, β=0.1 and $Br=1$ (isoflux case).

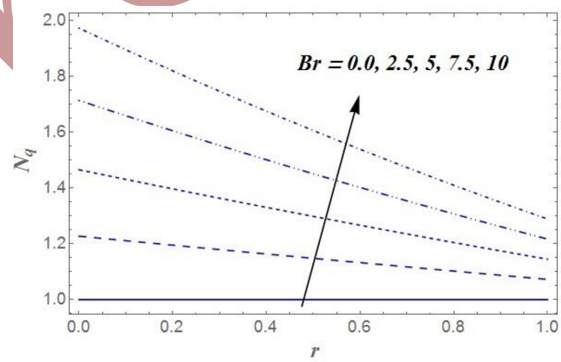

Fig. 7. Brinkman number influence on overall entropy generation plot for fluid with n=0.538, β=0.1 and $De=0.1$ (isoflux case).

Also, the characteristic parameters influence such as Deborah number and Brinkman number on Bejan number for isothermal case are shown in Figs. 8 and 9, respectively. The Bejan number range is close to zero, as depicted in Figs. 8 and 9, which represented the entropy generation due to fluid friction is dominated and entropy generation due to heat transfer is little contributed to the total entropy generation, in the other words major irreversibilities resulted from viscous dissipation effects. As shown in these figures, variation of Deborah number is uniformly affected overall domain of flow, but variation of Brinkmann number is much more affected the flow adjacent of boundary conditions (inner and outer cylinders), where the viscous dissipation effects are much more pronounced than other domain of flow. Recall the friction entropy generation of fluid is dominated mechanism in overall entropy generation.

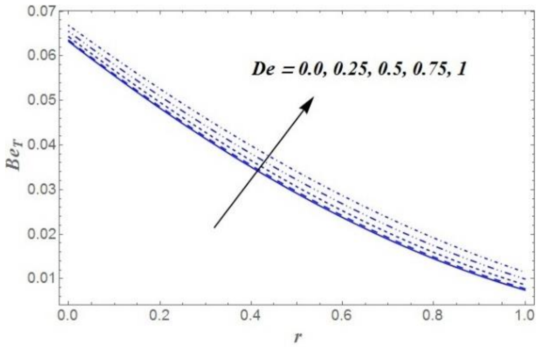

Fig. 8. Deborah number influence on Bejan number plot for fluid with *n*=0.538, β=0.1 and *Br=1* (isothermal case).

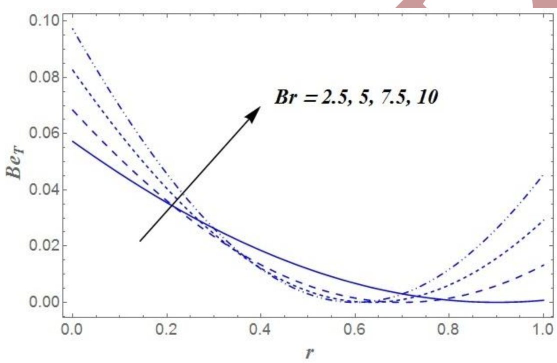

Fig. 9. Brinkman number influence on Bejan number plot for fluid with *n*=0.538, β=0.1 and *De=0.1* (isothermal case).

Next, the characteristic parameters influence such as Deborah number and Brinkman number on plot of Bejan number for Isoflux case are shown in Figs. 10 and 11, respectively. A similar trend can be presented for Bejan number plot in the radial direction from rotating inner cylinder to stationary outer cylinder (as is shown in Figs. 10 and 11). Although for Isoflux case, the Bejan number range is close to one, which represented the entropy generation due to heat transfer is dominated and entropy generation due to fluid friction is little contributed to the total entropy generation, in the other words major irreversibilities resulted from force convection heat transfer.

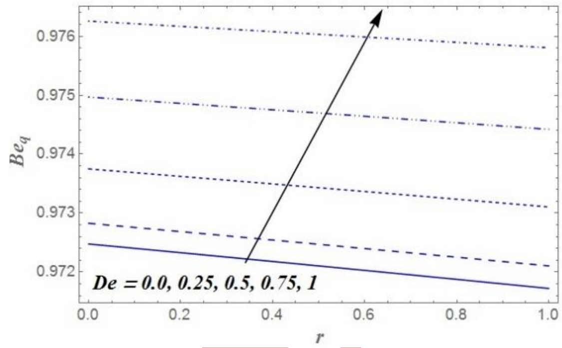

Fig. 10. Deborah number influence on Bejan number profile for fluid with *n*=0.538, β=0.1 and *Br=1* (isoflux case).

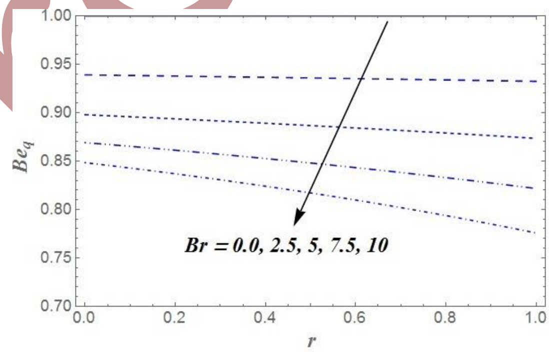

Fig. 11. Brinkman number on Bejan number profile in isoflux case for polystyrene solution with *n*=0.538, β=0.1 and *De=0.1*.

5. Conclusion

The entropy generation analysis is investigated for non-Newtonian fluid flow between concentric cylinders is examined when the inner cylinder is rotated at a specified angular speed and the outer cylinder is fixed. The non-Newtonian fluid viscosity is considered at the same time dependent on temperature and shear rate. Perturbation method is presented to construct analytical approximation expressions for entropy generation number in the rapidly convergent series form. The technique success for this problem it can be considered as a feasibility to use in other non-linear cases, instead of using other difficult and sophisticated techniques. It is concluded that the proposed technique is highly accurate. The influences of the Brinkman number as frictional dissipation parameter of flow and the Deborah number as elasticity parameter of fluid are investigated on the overall entropy generation analysis and Bejan number. The overall entropy generation number decays in the radial direction from rotating inner cylinder to stationary outer cylinder. The results show that overall entropy generation rate increases within flow domain as Brinkman number enhances. It, however, declines with enhancing Deborah number. The reason for this is very clear, the pseudoplastic fluid between concentric cylinders is heated as Brinkman number increases due to frictional dissipation and it is cooled as Deborah number increases which is due to the elasticity behavior of the fluid. For isothermal case, the results shown the entropy generation due to fluid friction is dominated and entropy generation due to heat transfer is little contributed to the total entropy generation, in the other words major irreversibilities resulted from viscous dissipation effects. Although for Isoflux case, the Bejan number range is close to one, which represented the entropy generation due to heat transfer is dominated and entropy generation due to fluid friction is little contributed to the total entropy generation, in the other words major irreversibilities resulted from force convection heat transfer. Therefore, Deborah number and Brinkman number must be controlled to minimize total entropy generation.

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