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Effect of Variable Thermal Conductivity and the Inclined Magnetic Field on MHD Plane Poiseuille Flow in a Porous Channel with Non-Uniform Plate Temperature

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Abstract

The aim of this paper is to investigate the effect of the variable thermal conductivity and the inclined uniform magnetic field on the plane Poiseuille flow of viscous incompressible electrically conducting fluid between two porous plates with Joule heating in the presence of a constant pressure gradient through non-uniform plate temperature. It is assumed that the fluid injection occurs at lower plate, while fluid suction occurs at upper plate. The governing equations of momentum and energy are transformed into coupled and nonlinear ordinary differential equations using similarity transformation and then solved numerically using finite difference technique. Numerical values for the velocity and temperature have been iterated by Gauss Seidal iteration method in Matlab programming to a suitable number so that the convergent solutions of velocity and temperature are considered to be achieved. Numerical results for the dimensionless velocity and the temperature profiles for different governing parameters such as the Hartmann Number (M) angle of inclination of magnetic field (α), suction Reynolds number (Re) Prandtl Number (Pr), Eckert number (Ec) and variable thermal conductivity (ε) have been discussed in detail and presented through graphs

Nomenclature

a	distance between two plates
B_0	applied magnetic field
$C_{1,}$ ...	constants
C_p	specific heat at constant pressure
Ec	Eckert number
h	increment along y-axis
K	thermal conductivity
K'	variable thermal conductivity
i	index refers to y
M	Hartmann number
m	number of grid points along y- direction
P	dimensionless pressure gradient
u_m	maximum velocity
v'	velocity along y'-direction
V	constant suction velocity along y'-direction
x', y'	cartesian coordinates
x, y	dimensionless cartesian coordinates

Greek letters

α	angle between velocity and applied magnetic field
ε	thermal conductivity parameter
θ'	fluid temperature

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p	fluid pressure	θ_0	temperature of the lower plate
Pr	Prandtl number	θ_1	temperature of the upper plate
p	pressure gradient	β	coefficient of thermal expansion
P	dimensionless pressure gradient	ρ	density of liquid
Re	suction Reynolds number	σ	electrical conductivity
u'	velocity along x'-direction	μ	coefficient of viscosity
u	dimensionless velocity along x-direction	υ	kinematic viscosity
θ	dimensionless fluid temperature	λ	magnetic diffusivity

1. Introduction

The study of magneto hydrodynamic (MHD) flow and heat transfer analysis in channel has been a topic of great interests by many researchers in the last few decades due to its wide range of applications in industries and engineering problems. Such applications are in solar technology, MHD power generators, MHD pumps, aerodynamics heating, electrostatic precipitation, purification of oil and fluid sprays and droplets, etc. Cooling process can be controlled effectively by theory of variable thermal conductivity for which the high quality product may be produced. Liquid metals having low Prandtl number are generally used as coolants because of its higher thermal conductivity and have applications in manufacturing processes such as the cooling of the metallic plate, nuclear reactor etc.

Palm et al. [1] studied on the steady free convection in porous medium and extended their work in an isotropic porosity with heat exchange effect. Bansal and Jain [2] studied the plane Poiseuille flow problem with unequal wall temperature of an incompressible fluid having temperature dependent viscosity. Arunachalam and Rajappa [3] presented an analysis of the steady laminar forced convection of liquid metals for the case of simultaneous linear variation in thermal conductivity and capacity. Chamkha [4] considered unsteady flow and heat transfer of an electrically conducting fluid through a porous medium channel in the presence of a transverse magnetic field. He found closed-form solutions for steady-state problem and numerical solutions for unsteady problem using implicit finite difference method. Chaim [5] studied heat transfer in fluid flow of low Prandtl number with variable thermal conductivity. Chamkha [6] studied the problem of unsteady laminar fully-developed flow and heat transfer of an electrically-conducting and heat-generating or absorbing fluid with variable properties through porous channels in the presence of uniform magnetic and electric fields. Nasrin and Alim [7] investigated the combined effects of viscous dissipation and temperature dependent thermal conductivity on MHD free convection flow with heat conduction and Joule heating along a vertical flat plate. Mahanti and Gaur [8] investigated the effects of linearly varying viscosity and thermal conductivity on steady free convective flow of a viscous incompressible fluid along an isothermal vertical plate in the presence of heat sink. Umavathi et al. [9] investigated the problem of unsteady oscillatory flow and heat transfer in a horizontal composite porous medium with viscous and Darcian dissipations. Manyange et al. [10] examined a steady MHD Poiseuille flow between two infinite porous plates in an inclined magnetic field. They considered the lower plate is assumed porous while the upper plate is not. Eegunjobi and Makinde [11] studied the effects of the thermodynamic second law on steady flow of an incompressible variable viscosity electrically conducting fluid in a channel with permeable walls and convective surface boundary conditions. Ceasar Muriuki et al. [12] investigated the magnetohydrodynamic (MHD) flow of a Newtonian fluid in a horizontal channel bounded by two parallel porous plates in the presence of inclined magnetic field. Kumar Jhankal and Kumar [13] studied the plane Poiseuille flow with unequal wall temperatures of an incompressible fluid with temperature dependent viscosity in the presence of transverse magnetic field is studied. Kuiry and Bahadur [14] investigated the magnetohydrodynamic behaviour of two dimensional Poiseuille flow under the influence of an inclined magnetic field and constant pressure gradient for a viscous, incompressible fluid between two infinite parallel plates of which the lower plate is taken to be porous. Uwanta and Usman [15] studied variable thermal conductivity and magnetic field intensity effects on heat and mass transfer flow over a vertical channel both numerically and analytically. Attia et al. [16] studied the unsteady flow in porous medium of a viscous incompressible fluid bounded by two parallel porous plates with heat transfer. A uniform and constant pressure gradient is applied in the axial direction whereas a uniform suction and injection are applied in the direction normal to the plates. Raju et al. [17] studied the steady MHD forced convective flow of a viscous fluid of finite depth in a saturated porous medium over a fixed horizontal channel with thermally insulated and impermeable bottom wall in the presence of viscous dissipation and joule heating. Deka and Basumatary [18] presented an analysis of the study the problem of the steady flow and heat transfer of an incompressible fluid over a static wedge in the presence of suction/injection with variable viscosity. Recently, Gupta et al. [19] discussed the effect of the variable thermal conductivity and the inclined uniform magnetic field on the plane Poiseuille flow of viscous incompressible electrically conducting fluid in the presence of a constant pressure gradient through non-uniform plate temperature. Yu et al. [20] investigated numerically natural convection flows of an electrically conducting fluid a under uniform magnetic field at different angles θ with respect to horizontal plane in rectangular cavities. Sai and Rao [21] studied the effects of suction or injection on an incompressible laminar flow in a rectangular duct with non-conducting walls in the presence of an imposed transverse magnetic field and obtained analytical solutions for velocity and magnetic field. Das and Jana [22] studied the effects of magnetic field and suction/injection on the entropy generation in a flow of a viscous incompressible electrically conducting fluid between two infinite horizontal parallel porous plates under a constant pressure gradient. Das and Jana [23] also studied the effects of Hall currents on entropy generation in a porous channel with suction and injection. Joseph et al. [24] investigated the effect of heat and mass transfer on unsteady MHD poiseuille flow between two infinite parallel porous plates in an inclined magnetic field. Idowu and Olabode [25] studied the unsteady MHD poiseuille flow between two infinite parallel plates in an inclined magnetic field with heat transfer. Ganesh and Krishnambal [26] investigated unsteady MHD Stokes flow of a viscous fluid between two parallel porous plates. They considered fluid being withdrawn through both walls of the channel at the same rate. Joseph et al. [27] studied the unsteady MHD Couette flow between two infinite parallel porous plates in the presence of an inclined magnetic field with heat transfer.

The aim of this study is to investigate the effect of variable thermal conductivity and the inclined uniform magnetic field on steady MHD plane Poiseuille flow through non-uniform plate temperature and with constant injection or suction and Joule heating. The governing differential equations for velocity and temperature are solved numerically by developing finite difference codes in Matlab programming.

2. Mathematical formulation

Consider steady viscous incompressible electrically conducting plane Poiseuille fluid flow bounded by two parallel porous plates separated by a distance a. The x'-axis is taken along the flow direction and the y'-axis is perpendicular to the plates. A uniform transverse magnetic field B_0 is applied normal to the plates and makes an angle α with flow direction. Both of the plates are kept stationary and it is assumed that the lower permeable plate at $y' = 0$, where fluid injection occurs maintained at constant temperatures Θ_0, while at $y' = a$, the upper permeable plate fluid suction occurs maintained at constant temperatures Θ_1, where $\theta_1 > \theta_0$. The flow is driven by the constant pressure gradient $\partial p/\partial x'$.

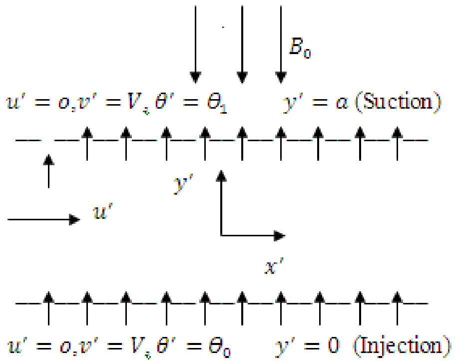

Fig.1: Geometry of the problem

It is assumed that the magnetic Reynolds number is very small, so that the induced electric field caused by induced magnetic field is assumed negligible. The flow in the region is unidirectional, steady laminar and fully developed so all the physical variables except pressure depend on y' only. The fluid particles are injected with velocity $v' = V$ at lower porous plate $y' = 0$ and sucked with the same velocity at the upper porous plate $y' = a$, so $\partial V/\partial y' = 0$. The governing equations of momentum and energy (Gupta et al., [19]) with Joule heating term are given by

$$V\frac{\partial u'}{\partial y'} = -\frac{1}{\rho}\frac{\partial p}{\partial x'} + \nu\frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2 \sin^2\alpha u'}{\rho} \quad (1)$$

$$\rho C_p V\frac{\partial \theta'}{\partial y'} = \frac{\partial}{\partial y'}\left(K'\frac{\partial \theta'}{\partial y'}\right) + \mu\left(\frac{\partial u'}{\partial y'}\right)^2 + \sigma B_0^2 \sin^2\alpha u'^2 \quad (2)$$

where, ρ is the density of the fluid, υ is the kinematic viscosity, σ is the electrical conductivity, μ is the co-efficient of viscosity, C_p is the specific heat at constant pressure, K' is the variable thermal conductivity, α $(0 \le \alpha \le \pi)$ is the is the angle between velocity and magnetic field strength, u' is the axial fluid velocity, p is the pressure and θ' is the fluid temperature.

The corresponding boundary conditions are

$$\left.\begin{array}{l} u' = 0,\ \theta' = \theta_0 \ \text{ at } \ y' = 0 \\ u' = 0,\ \theta' = \theta_1 \ \text{ at } \ y' = a \end{array}\right\} \quad (3)$$

Following Mahanti and Gaur [8], the thermal conductivity is considered to vary linearly with temperature and it is of the form

$$K' = K(1+\varepsilon\theta) \quad (4)$$

$v' = V$, constant suction velocity

Introducing dimensionless quantities as follows

$$y = \frac{y'}{a},\ u = \frac{u'}{u_m},\ \theta = \frac{\theta' - \theta_0}{\theta_1 - \theta_0} \quad (5)$$

Where $u_m = -\frac{a^2}{\mu}\frac{dp}{dx'}$, is the maximum velocity.

Since the flow is driven by a constant pressure gradient. It is sufficiently assumed that the maximum velocity ($u_m = -\frac{a^2}{\mu}\frac{dp}{dx'}$) contained in the middle of the channel in the plane Poiseuille flow with constant fluid properties (Schlichting, [28]).

Using (4) and dimensionless quantities (5), Eqs. (1) and (2) can be expressed as

$$\frac{d^2u}{dy^2} - \text{Re}\frac{du}{dy} - M^2\sin^2\alpha u + P = 0 \quad (6)$$

$$(1+\varepsilon\theta)\frac{d^2\theta}{dy^2} + \varepsilon\left(\frac{d\theta}{dy}\right)^2 - \text{Re}\,\text{Pr}\frac{d\theta}{dy} + Ec\,\text{Pr}\left(\frac{du}{dy}\right)^2 + Ec\,\text{Pr}\,M^2 sin^2\alpha u^2 = 0 \quad (7)$$

Where,

$\text{Re} = \frac{Va}{\nu}$, is the suction Reynolds number

$M = B_0 a\left(\frac{\sigma}{\nu}\right)^{1/2}$, is the Hartmann number

$\text{Pr} = \frac{\rho C_p}{\mu}$, is the Prandtl number and

$Ec = \frac{u_m^2}{C_p(\theta_1 - \theta_0)}$, is the Eckert number

$P = -\frac{a^2}{\mu u_m}\frac{dp}{dx'}$, is the dimensionless pressure gradient

Normalize boundary conditions are

$$\left.\begin{array}{l} u=0,\ \ \theta=0 \text{ at } y=0 \\ u=0,\ \ \theta=1 \text{ at } y=1 \end{array}\right\} \quad (8)$$

3. Numerical Solution

The coupled differential equations (6) and (7) subjects to boundary conditions (8) are solved using finite difference technique. In this method the derivative terms occurring in the governing differential equations have been replaced by their finite difference approximations. Central difference approximations of second order accuracy have been used because they are more accurate than forward and backward differences. Then, an iterative scheme is used to solve the linearized system of difference equations. The linearized system of equations based on our representing the step size by h, the finite difference equations corresponding to equation (6) and (7) are given as

$$\frac{u_{i+1}-2u_i+u_{i-1}}{h^2} - \mathrm{Re}\left(\frac{u_{i+1}-u_{i-1}}{2h}\right) - M^2 sin^2\alpha u_i + P = 0 \quad (9)$$

$$(1+\varepsilon\theta_i)\left(\frac{\theta_{i+1}-2\theta_i+\theta_{i-1}}{h^2}\right) + \varepsilon\left(\frac{\theta_{i+1}-\theta_{i-1}}{2h}\right)^2 - \mathrm{Re}\,\mathrm{Pr}\left(\frac{\theta_{i+1}-\theta_{i-1}}{2h}\right) + Ec\,\mathrm{Pr}\left(\frac{u_{i+1}-u_{i-1}}{2h}\right)^2 + Ec\,\mathrm{Pr}\,M^2 sin^2\alpha u_i^2 = 0 \quad (10)$$

On simplification of equations (9) and (10) for velocity u_i and temperature θ_i can be written as

$$u_i = C_1(u_{i+1}+u_{i-1}) - C_2(u_{i+1}-u_{i-1}) + C_3 \quad (11)$$

$$\theta_i = 0.5(\theta_{i+1}+\theta_{i-1}) + C_4(\theta_i\theta_{i+1} - 2\theta_i^2 + \theta_i\theta_{i-1}) - C_5(\theta_{i+1}-\theta_{i-1}) + C_6(\theta_{i+1}-\theta_{i-1})^2 + C_7(u_{i+1}-u_{i-1})^2 + C_8 u_i^2 \quad (12)$$

where,

$C_1 = \frac{1}{2+h^2M^2sin^2\alpha}$, $C_2 = \frac{h\,\mathrm{Re}}{2+h^2M^2sin^2\alpha}$, $C_3 = \frac{h^2P}{2+h^2M^2sin^2\alpha}$, $C_4 = 0.5\varepsilon$, $C_5 = \frac{h\,\mathrm{Re}\,\mathrm{Pr}}{4}$, $C_6 = \frac{\varepsilon}{8}$, $C_7 = \frac{Ec\,\mathrm{Pr}}{8}$ and $C_8 = \frac{Ec\,\mathrm{Pr}\,M^2sin^2\alpha}{2}$ are constants.

Then corresponding discretized boundary conditions take the form

$$\left.\begin{array}{l} u_i=0,\ \ \theta_i=0 \quad \text{at } i=1 \\ u_i=0,\ \ \theta_i=1 \text{ at } i=m+1 \end{array}\right\} \quad (13)$$

where i stands for y and m stands for number of grid points inside the computational domain considered.

4. Results and Discussion

In this investigation the effect of variable thermal conductivity and the inclined uniform magnetic field on steady MHD plane Poiseuille flow between two parallel porous plates through non-uniform plate temperature have been discussed numerically. Computational domain is divided into 100 uniform grid points. Numerical values for the velocity (u_i) and temperature (θ_i) have been iterated by Gauss Seidal iteration method in Matlab programming (Mathews and Fink [29]) to a suitable number so that the convergent solutions of u_i and T_i are considered to be achieved when the maximum differences between two successive iterations are less than a tolerance, 10^{-7} (Alkhawaja and Selmi [30]; Umavathi and Chamkha [31]) and computed results of velocity and temperature are presented in terms of graphics for relevant parameters such as Hartmann number (M), suction Reynolds number (Re), Parndtl parameter (Pr), Eckert number (Ec), dimensionless pressure gradient (P) and variable thermal conductivity (ε).

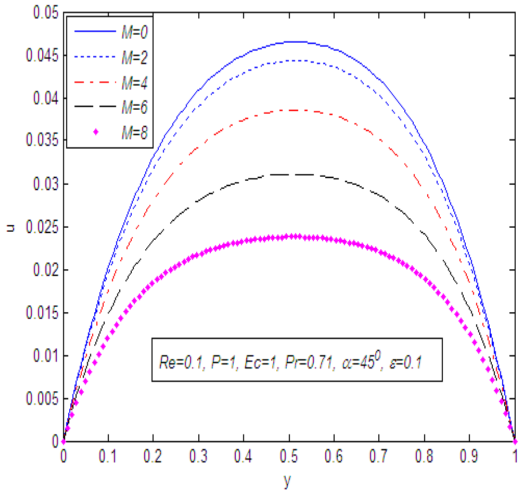

Fig. 2: Velocity profiles for different values of the Hartmann number M

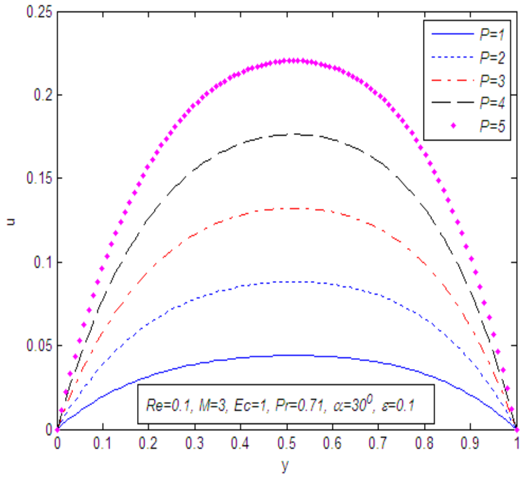

Fig. 5: Velocity profiles for different values of the dimensionless pressure gradient P

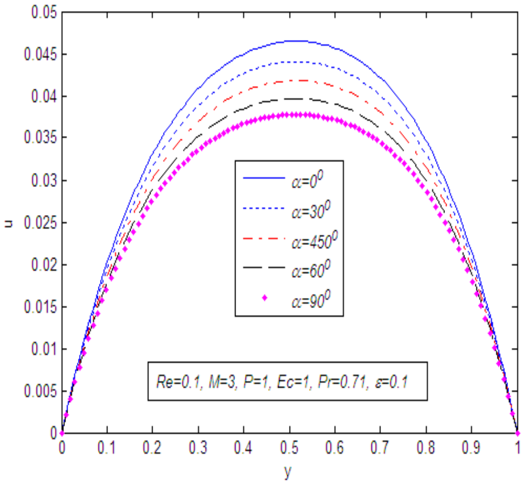

Fig. 3: Velocity profiles for different inclination angle α

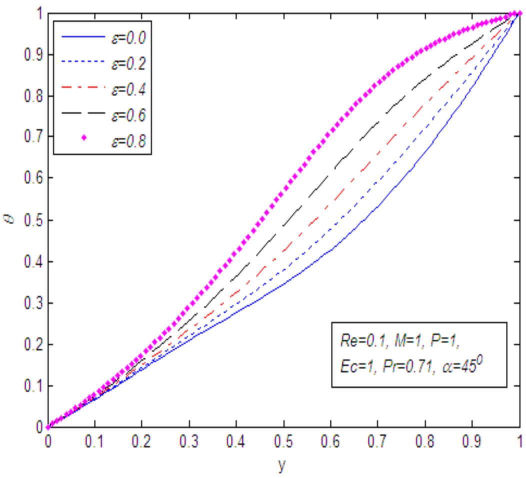

Fig. 6: Temperature field for different values of the variable thermal conductivity ε

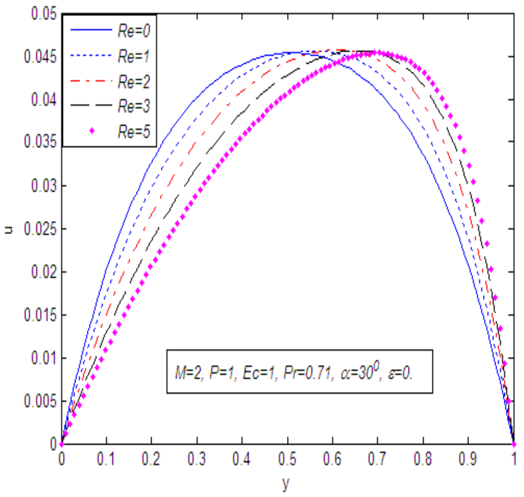

Fig. 4: Velocity profiles for different values of the suction Reynolds number Re

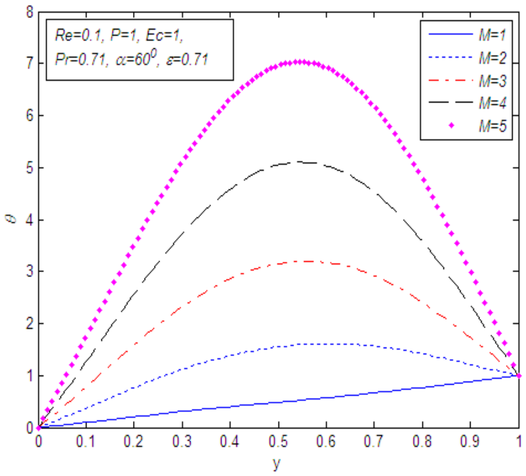

Fig. 7: Temperature field for different values of the Hartmann number M

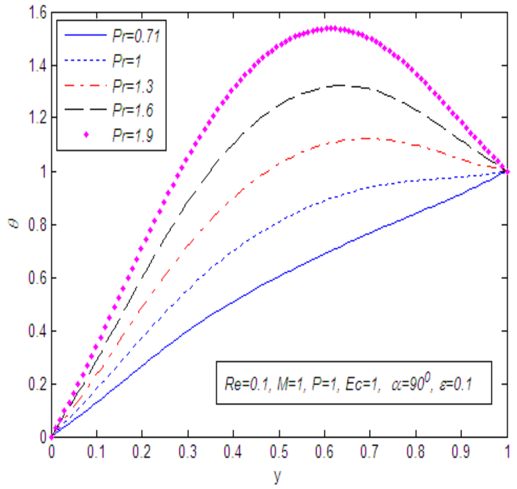

Fig. 8: Temperature field for different values of the Prandtl number *Pr*

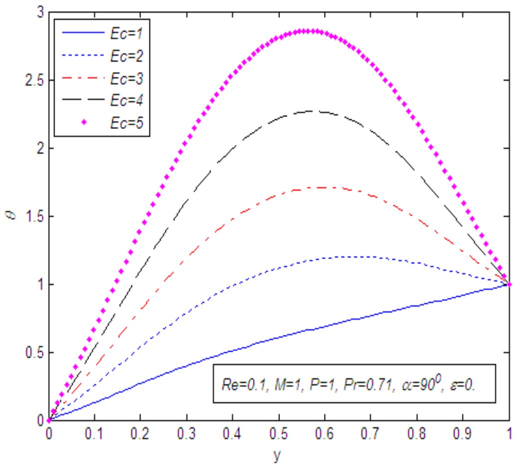

Fig. 9: Temperature field for different values of the Eckert number *Ec*

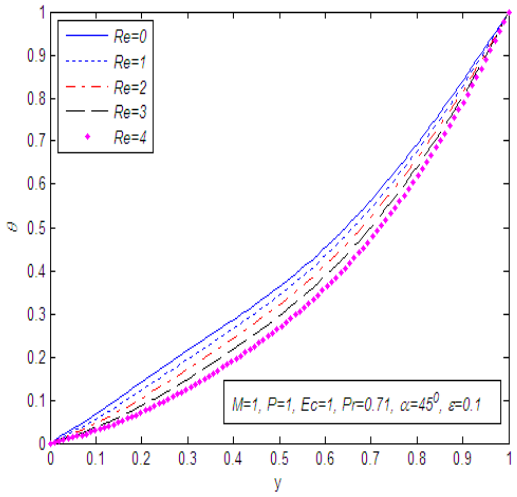

Fig. 10: Temperature field for different values of the suction Reynolds number *Re*

Effects of various dimensionless parameters on the velocity profiles are shown in Figs. 2–5, fixing other flow parameters constant. Usually, the velocity profiles are parabolic in geometries with zero values at the channel walls due to no slip condition and attain their maximum value in the middle portion of the channel for $Re \ll 1$. In Fig. 2, it is observed that the fluid velocity decreases with increasing Hartmann number M. This is due to fact that strength of the Lorentz force increases for increasing values Hartmann number, which acts as a retardation to the fluid flow.

Fig. 3 shows the effect of different inclination angle (α) on the velocity. It is evident that an increase in inclination angle (α), decreases the velocity of the flow field, maximum retardation of the velocity of flow occurs at the inclination angle $\alpha = 90^0$, signifies the intensity of Lorenz force is maximum which acts against the flow direction and has a tendency to slow down the motion of fluid flow. It confirms the analytical result obtained by Gupta et al. [19].

Fig. 4 presents the effect of suction Reynolds number Re on fluid velocity. It is observed that the fluid velocity is parabolic in geometry for $Re = 0$ (i.e without injection or suction), then decreases and skews towards the upper plate as suction Reynolds number $(Re > 0)$ increases due to increasing injection at the lower plate and increasing suction at the upper plate. This is good agreement with the result obtained by Eegunjobi and Makinde [11]; and Das and Jana [22].

The effect of dimensionless pressure gradient P on fluid velocity is shown in Fig. 5. It is obvious from the Fig. 5 that the velocity profile increases with increasing dimensionless pressure gradient. It establishes the results of Kuiry and Bahadur [14].

The effects of various dimensionless parameters on the temperature profile are demonstrated in Figs. 6-9, fixing other flow parameters constant.

The effect of variable thermal conductivity parameter (ε) on temperature field is shown in Fig. 6. It is observed that the variation of thermal conductivity affects temperature profiles. With the increase in the value of ε, the temperature of fluid increases.

Fig. 7 shows the influence of the Hartmann number (M) on the temperature field. As M

increases the fluid temperature increases within the channel. The enhancement of temperature may be due to the presence of Joule heating (or Lorentz heating) in the energy equation, which serves as additional heat source to the flow system.
The effect of Prandtl number Pr on temperature profile is depicted in Fig. 8. It is observed that fluid temperature increases for increasing values of Pr.
In Fig. 9, the effect of Eckert number Ec on temperature is depicted. It is noticed that temperature profile increases with the increasing value of Ec. This may be due to facts that as Ec increases the viscous heating increases due to increasing convective heating at the upper plate increases leading to a rise in the fluid temperature.
The effect of suction Reynolds number Re on temperature profile is shown in Fig. 10. It is observed that the temperature profile decreases at all the points of fluid flow for increasing values of Reynolds number (Re), It may be attributed to the fact that injection or suction procedure absorbs the heat.
Fig. 11 shows the effect of inclination angle (α) on temperature profile, it is noticed that fluid temperature increases as α increases.
The computed results obtained in terms of graphics in this investigation are compared with earlier analytical and numerical works of authors viz. Gupta et al. [19], Eegunjobi and Makinde [11] and Kuiry and Bahadur [14]; and Das and Jana [22]. It is observed that our results are in good agreement with the results of earlier works.

5. Conclusions

In this paper, the effect of the variable thermal conductivity and the inclined uniform magnetic field on the plane Poiseuille flow of viscous incompressible electrically conducting fluid between two porous plates in the presence of a constant pressure gradient through non-uniform plate temperature with Joule heating have been investigated. The non-linear differential equations for velocity and temperature are solved numerically by developing finite difference codes and using Gauss Seidal iteration scheme in Matlab software. Based on the computed results presented above in terms graphics, the following conclusions are made:

i. Fluid velocity decreases for both increasing Hartmann number (M) and inclination angle (α).
ii. Fluid velocity is parabolic in geometry for $Re = 0$ (i.e without injection or suction), then decreases and skews towards the upper plate as suction Reynolds number (Re$>$ 0) increases.
iii. Fluid velocity increases for increasing dimensionless pressure gradient (P).
iv. Fluid temperature increases as variable thermal conductivity (ε), Eckert number (Ec), Prandtl number (Pr), Hartmann number (M) and inclination angle (α) increases.
v. Fluid temperature decreases for increasing suction Reynolds number (Re).

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