



A comparison between modal damping ratios identified by NExT-ERA and frequency domain impact test

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Abstract

In this research, the modal parameters of a beam in free-free condition are extracted by performing different experiments in the laboratory. For this purpose, two different techniques are employed. The first methodology is considered as a time domain method in Operational Modal Analysis. The other one is frequency domain impact hammer test which is categorized as an Experimental Modal Analysis method and can be regarded as the most common method in modal analysis. Checking the results obtained by the two methods, one can notice a distinct inconsistency in modal damping ratios extracted by each method. However, based on recent publications on the subject, it can be inferred that the time domain methods have better accuracy in identifying damping ratios of structures. In order to confirm the findings, the effect of excitation is examined for each method by altering the excitation tool. For the operational method, it is concluded that changing the excitation tool will not have a noticeable influence on the identified damping ratios, whilst for the Experimental Modal Analysis method changing the hammer tip leads to inconsistent results for damping ratios. This study exemplifies the deficiency of Experimental Modal Analysis methods in their dependency on excitation techniques.

1. Introduction

Modal analysis is considered as a customary method for identifying the dynamic properties of structures such as natural frequencies, damping ratios and mode shapes which can later be used to develop mathematic models of dynamic systems. In this part, fundamental subjects in the modal analysis will be elaborated, and after a

brief literature review, the main points of this research will be presented.

1.1. Time domain and frequency domain methods

Methodologies in modal identification are divided into two major categories which are time domain and frequency domain methods. Frequency domain methods use Fast Fourier

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Transform (FFT) for the recorded response and excitation data; therefore, the modal parameters are extracted in the frequency domain. On the other hand, time domain methods, which are developed based on concepts of modern control theory and computer engineering, use raw and direct structure response data so that the problems, related to Fourier transform such as leakage and aliasing, are avoided. [1]

Time domain methods maintain many advantages over their frequency domain counterparts and this fact is the main motivation for this study. For example, there is no need to record the excitation data in time domain methods; therefore much less computer memory is used in the analysis. Besides, time domain methods have been widely utilized for damage diagnosis purposes. The frequency domain methods, to which more attention has been paid in the classic modal analysis, employ filters and windows to the structure response data that cause a deficiency in extracting accurate modal damping ratios [2]. In other words, these filters and windows induce undesirable damping to the structure. This problem is much more critical for cases in which the structures possess low damping values. Furthermore, frequency domain methods establish Frequency Response Function (FRF) curves that usually exhibit errors in identifying the closely-spaced modes of structures. Due to their disinterest in employing any filter or window as well as their independence from FRF curves, time domain methods are considered as better candidates for studying structures with closely-spaced modes. It has also been reported that time domain methods entail better accuracy for identifying the low-frequency modes [3]. However, due to their computational simplicity, frequency domain methods have gained more attention in the classic modal analysis.

1.2. Experimental and operational modal analysis

In Experimental Modal Analysis (EMA), which is sometimes referred to classic modal analysis, there is an urge to measure the excitation signals along with the structure response data to compute the modal parameters of

systems. Different algorithms such as Single-Input Single-Output (SISO), Single-Input Multi-Output (SIMO) and Multi-Input Multi-Output (MIMO) were developed in time, frequency and space domains. However, there are limitations in EMA methods that beckon the need for improvement. For example, in these methods, the excitation is artificial and usually consists of hammer impacts or shaker loads that act through a single point. Therefore, the tests are only limited to be performed in a laboratory. Moreover, the applied artificial loads cannot be considered as feasible excitations for large structures in which all Degrees of Freedom (DOF) will not be excited by the load exerted in a single point. This limitation also includes complicated structures consisting of several smaller parts, since the whole structure cannot be studied in one step. In these cases, single parts should be tested independently while the boundary conditions are simulated by reasonable assumptions which are added to the modeling complexity.

In the early 1990s, Operational Modal Analysis (OMA) attracted the attention of civil engineers for use in cases such as ocean platforms, buildings, towers, bridges, etc. In OMA which is also referred to ambient and natural excitation or output-only modal analysis, there is no need to measure the excitation signals. The modal identification is performed by only using the response data of structure excited by natural ambient loads in real operational condition. OMA has been widely used for practical purposes in aerospace and mechanical engineering, as well. The methods entail a lot of advantages over their EMA counterparts. For example, the OMA methods are much faster and cheaper. In these methods, there is no need for expensive excitation tools and measuring their signals. The actual loading of the structure, in its practical condition, can be regarded as the excitation. Besides, for OMA studies the whole structure including its all detailed parts can be surveyed in one step and the boundary condition simulation is not required. Furthermore, because of the broadband excitation exerted at different DOFs, the detection of all vibration modes is assured.

Natural Excitation Technique (NExT) is one of the most prominent OMA methods in the time domain which can be combined with the MIMO time domain methods in EMA such as Polyreference Complex Exponential (PRCE), Extended Ibrahim Time Domain (EITD), and Eigensystem Realization Algorithm (ERA) [4].

1.3. Literature review

Performing a comparison between time domain and frequency domain methods is one of the freshest and prospering topics in modal analysis. Many studies have been conducted in which the modal parameters extracted by different methods are compared. Juang and Pappa [5] developed an ERA to identify the modal parameter and model reduction for dynamical systems. Le and Paultre [6] proposed the time–frequency domain decomposition method for the modal identification of ambient vibration testing. The modal parameter estimation of time-varying structures under unknown excitation is investigated by Zhou et al. [7]. Barjic et al. [8] studied the accuracy of ERA, ITD, and PRCE for OMA case and compared their obtained damping ratios for numerical systems. Gomaa et al. [9] compared the damping ratios extracted by Frequency Domain Decomposition (FDD) and Enhanced FDD as well as Stochastic Subspace Identification (SSI). It was concluded that the damping ratios identified by SSI, a time domain OMA method, was half those identified by the frequency domain method. Comparison between the modal damping ratios achieved by considering the measurement duration, frequency range, sampling rate, and the method used in modal parameter identification was studied by Kudu et al [10]. Again Brincker et al. [11] compared the damping ratios identified by ERA and FDD, and it was concluded that ERA gives better damping estimations. The Ibrahim Time Domain algorithm has been introduced to identify modal parameters from measurements in the OMA by Mohanty and Rixen [12]. Magalhaes et al. [13] studied the accuracy of FDD and SSI for the case of close modes. Mohanty and Rixen also proposed a modified ERA method where the harmonic excitation

components are explicitly taken into account during the identification [14].

Pintelon and et al. presented a numerical stable method, for calculating of uncertainty bounds on the estimated modal parameters in operational modal analysis [15].

Use of the NExT and the ERA models for output-only modal identification of civil infrastructure was done by Caicedo [16]. Implementation of the natural excitation technique (NExT) combined with the ERA to determine the dynamic characteristics of the aircraft structure was studied by Moncayo et al. [17]. Identification of modal parameters from multivariable transmissibility measurements is investigated by Devriendt et al [18].

In Agneni's study [19] the simple FRF based impact test results were compared with the ones identified by FDD and Hilbert Transform Method (HTM). A new operational modal identification method, frequency–spatial domain decomposition (FSDD) is developed by Zhang et al. [20]. Chen et al. [21] studied the validity of results identified by FDD and SSI. In their study, the damping ratios were computed by the time domain method only. Pioldi and Rizzi suggested the refined Frequency Domain Decomposition algorithm and an improved data-driven Stochastic Subspace Identification (SSI-DATA) procedure, working in the Frequency Domain and in the Time Domain, respectively to estimating modal dynamic properties of buildings under earthquake excitation [22].

Based on these studies it can be concluded that the time domain methods, especially ERA, result in more accurate damping ratios for structures. However, most of the studies used ERA method for numerically simulated systems and so far few studies have been conducted to survey NExT-ERA in experimental cases.

In this paper, the comparison has been made between OMA in the time domain, and EMA in the frequency domain using impact hammer test to determine the dynamic characteristics of structures. In order to assure the validity of ERA and NExT methodology, evaluation has been performed using finite element analysis. To show the correlation between the identified mode shapes, the Modal Assurance Criterion

(MAC) diagram is plotted for the mode shapes recognized by these two methods and also for OMA and EMA ones. The damping ratios estimated by the time domain OMA method ERA in conjugation with NExT is compared with the ones identified by the frequency domain impact hammer test in the experimental condition, which is the innovation of this study. Moreover, the effect of hammer tip is investigated for both methods. The results obtained in this section are also novel and have not been reported in previous works.

2. Discussion of methods

In this research, the time domain method for the ambient loading case is NExT which can be combined with MIMO algorithm in EMA such as PRCE, EITD, and ERA. Based on the recent investigations [23, 24] it can be concluded that ERA gives better results in comparison with the other two methods. Thus, in this research ERA is exploited in conjugation with NExT to extract modal properties in case of ambient loadings.

2.1 Eigensystem realization algorithm

The ERA is one of the most prominent modal identification techniques in the time domain which can be used in MIMO cases. The method is developed based on concepts in control theory. In this section, a brief review of the formulation for ERA as well as its algorithm is presented. In this method, the dynamic equation of a system is written in the way shown in Eq. (1)

$$[M]\{\ddot{y}(t)\} + [C]\{\dot{y}(t)\} + [K]\{y(t)\} = \{f(y(t), t)\} \quad (1)$$

in which the vector $\{y(t)\}$ is of dimension N and represents the displacement for different DOFs of the structure. In the state space, $\{u(t)\} = \begin{Bmatrix} y(t) \\ \dot{y}(t) \end{Bmatrix}$ can be defined in which $\{u(t)\}$ has a dimension of $2N$. In this method, the structure response for the measured coordinates at k^{th} time sample is shown by $\{x(k)\}$, which is a vector of dimension p , the number of output coordinates.

Vectors $\{u(t)\}$ and $\{x(k)\}$ are related by the matrix $[R]$ which is called the transformation matrix and is evolved during the transformation to the state-space. In ERA algorithm, for $\{u(k)\}$ which is in fact $\{u(k\Delta t)\}$, the following equation is proved to be valid:

$$\{u(k)\} = [A]\{u(k-1)\} + [B]\{\delta(k-1)\} \quad (2)$$

The Matrices $[A]$ and $[B]$ are called state-space matrices. Besides, $\{\delta(k)\}$ is a vector of dimension q and delineates the impacted coordinates at k^{th} time sample. For this method, the Markov parameter can be defined by Eq. (3).

$$[X(k)] = \begin{bmatrix} x_{11}(k) & x_{12}(k) & \dots & x_{1q}(k) \\ x_{21}(k) & x_{22}(k) & \dots & x_{2q}(k) \\ \vdots & \vdots & \ddots & \vdots \\ x_{p1}(k) & x_{p2}(k) & \dots & x_{pq}(k) \end{bmatrix} \quad (3)$$

In this matrix, x_{ij} is defined as the Impulse Response Function (IRF) of i^{th} coordinate due to the impact acted at j^{th} coordinate. In the algorithm, it is assumed that the impact has occurred at time 0 while the structure is in the stationary position (zero initial condition). Implementing these assumptions in Eq. (2) yields:

$$\underbrace{[X(k)]}_{p \times q} = \underbrace{[R]}_{p \times 2N} \underbrace{[A]^{k-1}}_{2N \times 2N} \underbrace{[B]}_{2N \times q} \quad (4)$$

At this stage, the realization algorithm begins based on which the matrices $[A]$, $[B]$ and $[R]$ are defined so that the order of the corresponding dynamic system is minimum. Due to the mathematical and logical complexity of the realization algorithm, here only a summary of the procedure is presented. The interested readers can access the detailed proof for obtaining the state-space matrices in reference [25].

For the identification process, the Hankel matrix is generated as in Eq. (5).

$$\underbrace{H(k-1)}_{pr \times qs} = \begin{bmatrix} [X(k)] & [X(k+1)] & \dots & [X(k+s-1)] \\ [X(k+1)] & [X(k+2)] & \dots & [X(k+s)] \\ \vdots & \vdots & \ddots & \vdots \\ [X(k+r-1)] & [X(k+r)] & \dots & [X(k+r+s-2)] \end{bmatrix} \quad (5)$$

For this matrix, r and s are two positive integers for which the rank of Hankel matrix reaches a stationary value. Employing the Singular Value Decomposition for the $[H(0)]$ yields:

$$\underbrace{[H(0)]}_{pr \times qs} = \underbrace{[U]}_{pr \times pr} \underbrace{[\Sigma]}_{pr \times ps} \underbrace{[V]^T}_{ps \times qs} \quad (6)$$

It can be verified that the rank for $[H(0)]$ is $2N$. In other words, the matrix entails $2N$ singular values, a number which is equal to the order of the corresponding state-space system. $[H(0)]$ can also be computed by taking only the first $2N$ columns of matrices $[U]$, $[\Sigma]$ and $[V]$ into account:

$$\underbrace{[H(0)]}_{pr \times qs} = \underbrace{[U_{2N}]}_{pr \times 2N} \underbrace{[\Sigma_{2N}]}_{2N \times 2N} \underbrace{[V_{2N}]^T}_{2N \times qs} \quad (7)$$

in which $[U_{2N}]^T [U_{2N}] = [V_{2N}]^T [V_{2N}] = [I]$. The matrices $[E_p]$ and $[E_q]$ are defined as Eqs. (8 and 9).

$$\underbrace{[E_p]^T}_{p \times pr} = \begin{bmatrix} [I] & [0] & \dots & [0] \end{bmatrix} \quad (8)$$

$$\underbrace{[E_q]^T}_{q \times qs} = \begin{bmatrix} [I] & [0] & \dots & [0] \end{bmatrix} \quad (9)$$

Finally, complicated mathematical operations, which are avoided here, conclude that matrices $[A]$, $[B]$ and $[R]$ can be computed as follows:

$$[R] = \left[[E_p]^T [U_{2N}] [\Sigma_{2N}]^{\frac{1}{2}} \right] \quad (10)$$

$$[B] = \left[[\Sigma_{2N}]^{\frac{1}{2}} [V_{2N}]^T [E_q] \right] \quad (11)$$

$$[A] = \left[[\Sigma_{2N}]^{-\frac{1}{2}} [U_{2N}]^T [H(1)] [V_{2N}] [\Sigma_{2N}]^{-\frac{1}{2}} \right] \quad (12)$$

Eigenvalue solution of the matrix $[A]$ can be utilized to identify the modal properties of the system. Therefore, $2N$ complex conjugate eigenvalues is computed.

$$[A]\{\Psi_u\} = \lambda\{\Psi_u\} \quad (13)$$

The obtained values for λ will be of the form shown in Eq. (13). Subsequently, the natural frequencies and damping ratios can be computed as Eqs. (14)- (16).

$$\lambda_r = e^{s_r \Delta t} = e^{(s_r^R + i s_r^I) \Delta t} \quad (14)$$

$$\omega_r = \sqrt{(s_r^R)^2 + (s_r^I)^2} \quad (15)$$

$$\zeta_r = -\frac{s_r^R}{\sqrt{(s_r^R)^2 + (s_r^I)^2}} \quad (16)$$

Furthermore, the $2N$ mode shapes for the system can be identified by Eq. (17) [18].

$$\underbrace{\{\Psi_x\}}_{p \times 1} = \underbrace{[R]}_{p \times 2N} \underbrace{\{\Psi_u\}}_{2N \times 1} \quad (17)$$

2.2. Natural excitation technique

As it was stated in the previous chapter, NExT is one of the OMA methods in the time domain. In this method Correlation Function (COR) for the natural excitation (ambient loading or white noise) can be written as the summation of sinusoids so that the modal parameters i.e. natural frequency, modal damping ratio and mode shape coefficient for each sinusoid is equal to those for the corresponding structural dynamic mode. Hence, MIMO time domain methods in classic modal analysis can be utilized to extract the modal properties of structures in a way that COR is used instead of IRF.

For NExT method, $R_{ij}(t)$ is the COR for the random response at i^{th} and j^{th} coordinate at time t can be treated as the response at i^{th} coordinate due to the impulse acted at j^{th} coordinate. For discrete time response data, COR is computed for the random response at two coordinates as shown in Eq. (18).

$$R_{ij}(m) = \frac{1}{N} \sum_{n=1}^{N-m+1} x_i(n) x_j(x+m-1) \quad (18)$$

where N is the total number of recorded data. Increasing N can lead to a better accuracy in computing COR.

3. Numerical simulation

Based on the discussion provided in the previous chapter, MATLAB codes are developed for ERA and NExT. In order to assure the validity of methodology as well as the well-development of codes, evaluation study is performed for a numerical system before entering the laboratory and performing experimental tests. For this purpose, a finite element (FE) model of a 1 m long cantilever beam with a square 2×2 cm cross section is established in commercial software. First, the actual modal properties are extracted by a simple modal solution. Next, random successive impulses are modeled and inserted at several points of structure, and the response is recorded for the numerical solution of the software, a damping ratio of 0.6 is assigned for all vibration modes while the corresponding time steps are set 0.0002. Yet the response is recorded in a one-in-between way which delineates a sampling frequency of 2500 Hz. Besides, 10000 acceleration data is used for the identification process. The modal parameters extracted by NExT-ERA are presented in Table 1.

The MAC diagram is also plotted for the mode shapes identified by FE solution and those identified by NExT-ERA. The diagram is depicted in Fig. 1 which shows an adequate correlation between the identified mode shapes. For the FE simulation, the structure is divided into 100 elements resulting in 100 nodes. In the first step of modal identification, the response at all 100 nodes is considered in the modal analysis.

Table 1. Modal parameters extracted by NExT-ERA for the FEM beam.

Mode number	Natural frequency (Hz)	Error (%)	Damping ratio (%)	Error (%)
1	16.7629	0.02	0.6003	0.05
2	104.8609	0.00	0.6000	0.00
3	292.9072	0.00	0.6000	0.00
4	572.0130	0.00	0.6000	0.00

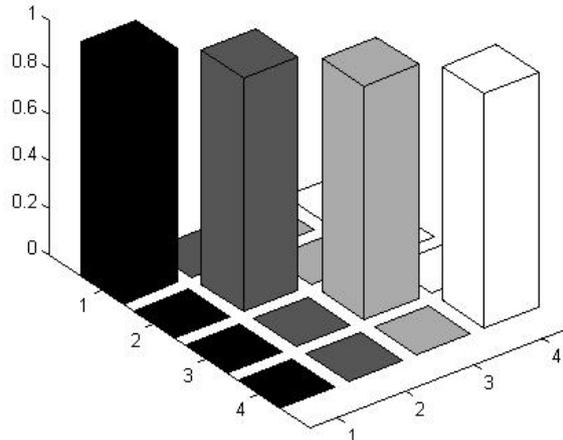


Fig. 1. MAC diagram for mode shapes identified by NExT-ERA for the FEM beam.

However, for practical experiment cases, such number of output coordinates for a beam does not seem feasible. Thus, investigations must be performed to recognize the number of required output coordinates in the real experiment. For the case of five output coordinates the computed modal properties are exhibited in Table 2.

Table 2. Modal parameters extracted by NExT-ERA for the FEM beam using only five output coordinates.

Mode number	Natural frequency (Hz)	Error (%)	Damping ratio (%)	Error (%)
1	16.7683	0.05	0.6010	0.17
2	104.8596	0.00	0.6002	0.04
3	292.8981	0.00	0.6007	0.11
4	572.1626	0.03	0.6013	0.21

It can be concluded that for this number of output coordinates the obtained results are acceptable since a slight drop has occurred in the accuracy of estimated damping ratios only. Therefore, assuming five output coordinates for the beam seems appropriate. For the new case, the MAC diagram is the same as the previous one; consequently, its dual representation is avoided here.

4. Experimental test

In this section, the modal parameters of a steel beam in free-free boundary condition are extracted. The beam has a rectangular 1×5 cm

cross-section and a 1 m length. First, the modal identification is performed by the OMA time domain method i.e. NExT-ERA. A simple impact hammer test in the frequency domain is also performed to compare the results. Six points are highlighted on the beam to be considered as the output coordinates which seems to be a reasonable number regarding the results obtained in the numerical investigation. In order to acquire the response data for NExT-ERA, six accelerometers are needed to be concurrently attached to the structure. However, for lightly-weighted structures, the attachment of several accelerometers does not seem reasonable since the added mass can cause an alteration in modal parameters of the structure. It would be much better to perform the test with only one accelerometer as it is for the hammer test where the test is performed in six steps. But, in this way there is no consistency in the obtained results (especially mode shapes) since the excitation, due to its random nature, does not remain the same for each step. To resolve the issue two accelerometers are usually used for OMA tests in which one accelerometer is fixed at one coordinate and the other is shifted through the remaining coordinates in separate steps (Fig. 2). For the beam model, the test is performed in five steps. At each step, the structure is excited by random hammer impacts while the response is being recorded (Fig. 3).



Fig. 2. Setup of OMA test.

PCB accelerometer sensors are used for the test as well as a four-channel NI analyzer unit while

the Lab View software is run for data acquisition. To assure the obtained results each test is conducted two times. For the identification using NExT-ERA, 20000-time history acceleration data is acquired by a sampling frequency of 2000 Hz. The results obtained by the hammer test, together with the ones extracted by NExT-ERA, are displayed in Table 3. In this table, a steel hammer tip for the impact test is used while the damping ratios are estimated by Polymax method. The MAC diagram for the mode shapes is plotted in Fig. 4 which reveals the well-correlation within the results obtained the two methods.

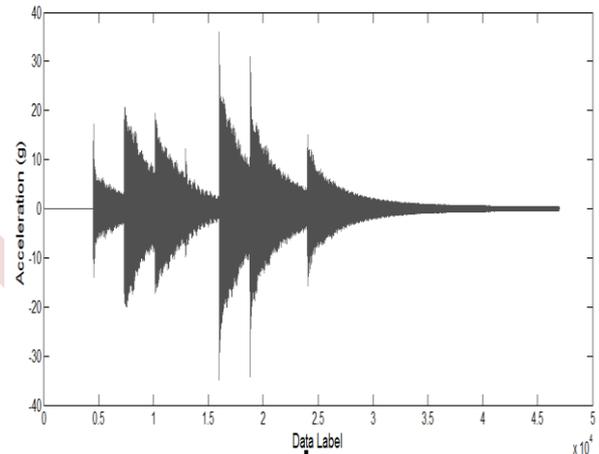


Fig. 3. An example of acquired data for NExT-ERA.

Table 3. Modal parameters extracted by OMA method and impact hammer test.

Mode	Natural frequency (Hz)		Damping ratio (%)	
	Impact test	NExT-ERA	NExT-ERA	Impact test
1	52.2793	52.2356	0.0424	0.370
2	143.939	143.6273	0.0946	0.175
3	282.358	281.5629	0.0386	0.091
4	466.828	465.1259	0.0213	0.045

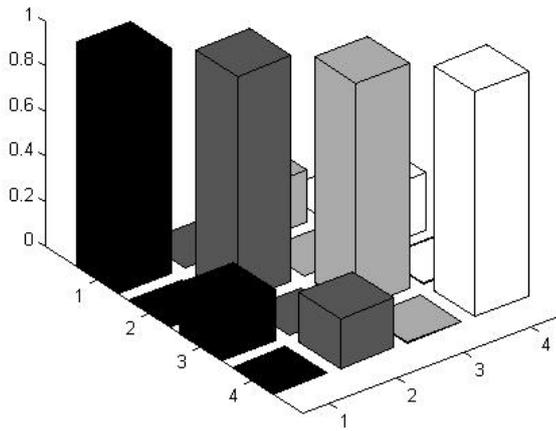


Fig. 4. MAC diagram for mode shapes identified by NExT-ERA and impact hammer test.

4.1. Hammer tip effect

As shown in Table 3 there is much contradiction between the damping ratios obtained by NExT-ERA and those identified by the hammer test. Viewing the results reported in references [9], [11] and [13], one can conclude that the damping ratios identified by NExT-ERA are more reliable since it is a time domain method. Besides, as it has been stated before, for the results presented in Table 3, steel hammer tip is used to perform the excitation in the impact test. To study the effect of hammer tip for the FRF-based impact test, three individual hammer tests are performed with variable hammer tips. It is noticed that the variation in hammer tip does not change the natural frequencies and the mode shapes identified by the impact test. However, there is a clear inconsistency in the damping ratios obtained by each test, as it is expressed in Table 4. For this table, the damping ratios are identified by Polymax method, as well.

Table 4. Effect of hammer tip on the damping ratios extracted by the impact hammer test.

Mode	Rubber tip	Plastic tip	Steel tip
1	1.24	0.72	0.37
2	0.40	0.30	0.23
3	0.11	0.11	0.10
4	0.06	0.05	0.05

The contradiction demonstrated in Table 4 proves the damping ratios identified by the hammer test deficient. This, in fact, reveals the influence of excitation tool on the results

obtained by EMA methods which is one of their major limitations. While damping ratio is an intrinsic characteristic of structures and does not depend on the excitation.

In order to ensure the results given by the OMA method, the same procedure is repeated for NExT-ERA. Three separate tests are performed and for each experiment, a different hammer tip is used for excitation. The results are presented in Table 5. As it can be seen changing the hammer tip does not affect the damping ratios computed by NExT-ERA noticeably, which proves the OMA method independent from the excitation tools.

Table 5. Effect of hammer tip on the damping ratios extracted by NExT-ERA.

Mode	Rubber tip	Plastic tip	Steel tip
1	0.0424	0.0413	0.0439
2	0.0946	0.0939	0.0951
3	0.0386	0.0391	0.0378
4	0.0213	0.0229	0.0224

5. Conclusions

In this research, a time domain OMA method, NExT-ERA, is investigated. The obtained results are compared with the ones identified by the most common method in the modal analysis which is the FRF-based impact test. It is concluded that there is not any notable difference in the natural frequencies and mode shapes extracted by the methods. The damping ratios identified by each method, however, contrast dramatically. The modal damping ratios computed by NExT-ERA are quite half the ones identified by hammer test. Referring the recent studies, and taking into account the discussion made in the introduction part of the paper, the weakness of frequency domain test in computing precise damping ratios can be concluded. In other words, the windows and filters applied to the structure response in frequency domain methods cause errors in computing the exact values for damping ratios. Moreover, for the FRF-based impact test, changing the hammer tip leads to severe changes in estimated damping ratios, which demonstrates the weakness of EMA methods in their dependency on the excitation tool. It is worth mentioning that for the

method, variation in hammer tip does not lead to inconsistency in estimating the natural frequencies and mode shapes. While for the OMA method no inconsistency happens even in computing the modal damping ratios. This reveals that the OMA methodology does not rely on the excitation tool for modal identification.

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