



# LPV Control for Speed of Permanent Magnet Synchronous Motor (PMSM) with PWM Inverter

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## ABSTRACT

This paper deals with the modeling, analysis, design and simulation of a robust control method for a permanent magnet synchronous machine (PMSM) supplied with a PWM inverter based on a Linear Parameter Variation (LPV)  $H_\infty$  standard controller. Under the influence of uncertainties and external disturbances, by a variation of  $\pm 150\%$  of motor parameters from the nominal values, the robust performance control problem is formulated as a LPV  $H_\infty$  standard scheme and solved by a suboptimal Linear Matrix Inequality (LMI) iterative  $H_\infty$  strategy. This new design method is able to ensure the stabilization of the augmented system formed of the perturbed system with improved performance in face of parameter variation and external disturbances. A Simulation study was carried out to illustrate the effectiveness of the proposed method. The results simulation are compared with a simple conventional PI controller.

## 1. INTRODUCTION

In recent years, growth in magnetic power device, and control theory have made the Permanent Magnet Synchronous Machine (PMSM) play an important role in high performance industrial drives at medium power range. Solar arrays of satellite are the main part of supplying energy for spacecraft. The damper is part of the deployment system of solar array on satellites which is designed to provide the damping required to control the deployment speed of the spring driven solar array. Because of the restriction of structure mass and material performance, the passive damper cannot satisfy the demand. Active damper is a promising choice due to its high dynamic performance. The active PMSM-based damper is used to release the impact on satellite attitude brought by solar array deployment [1].

The desirable idiosyncrasy of the PMSM is its compact simple structure, high-energy efficiency, high

air-gap flux density, reliable operation, high torque capability and high power density. Compared to induction motors, PMSM has the advantage of higher efficiency, due to the absence of the rotor losses and lower vacuum current below the rated speed. Also, its decoupling control performance is much less sensitive to the parameter variations of the motor [2]. Nevertheless, the control performance of the PMSM model is still influenced by the uncertainties of the plant, which are composed of unpredictable plant parameter variations and external disturbances. Thus, many approaches such as optimal control [3], adaptive control [4], [5], nonlinear control [6] and robust control [7] have been developed for the PMSM to deal with uncertainties. During the past decade, the  $H_\infty$  control problem has been extensively celebrated for its robustness in counter-acting parameters variations and external disturbances. The main objective of the  $H_\infty$  control is to synthesize a controller making the closed-loop system to satisfy a

prescribed  $H_\infty$  norm constraint, representing desired stability or tracking requirements. However, to ensure good performance under uncertainty perturbations, the  $H_\infty$  approach give usually a solution with high control gain. Motivated by this idea, our main contribution in this paper, is to develop an efficient robust method, called Linear Parameter Variation (LPV)  $H_\infty$  control that is insensitive to these variations.

The organization of the paper is as follows. First, the problem notification is presented and the nominal and uncertain plant of the PMSM is described. Then, a robust LPV  $H_\infty$  controller is developed. Besides, Linear Matrix Inequalities (LMIs) incorporated in the system to ensure  $H_\infty$  performance are mentioned. Finally, the control law design is carried out to the regulation of the speed and d-axis current of the PMSM.

## 2. STATE MODEL OF THE PERMANENT MAGNET SYNCHRONOUS MACHINE (PMSM)

The used PMSM is fed by a two-level voltage inverter. To obtain non-alternative quantities and considering conventional assumption, the mathematical model in the rotor reference of the PMSM can be represented by the following set of equations [8]:

$$v_q = Ri_q + \dot{\lambda}_q + \omega_s \lambda_d \quad (1)$$

$$v_d = Ri_d + \dot{\lambda}_d - \omega_s \lambda_q \quad (2)$$

$$\dot{x} = Ax + B_u u + B_w w$$

where

$$\lambda_q = L_q i_q \quad (3)$$

$$\lambda_d = L_d i_d + L_{md} I_{fd} \quad (4)$$

$$\omega_s = p \omega_r \quad (5)$$

$i_d$  and  $i_q$  are the d-q axis stator currents,  $v_d$  and  $v_q$  are the d-q axis stator voltages,  $R$  is the stator resistance,  $L_d$  and  $L_q$  are the d-q axis stator inductances,  $\omega_r$  is the motor speed and  $p$  is the number of poles. Torque equations are as

$$T_e = 1.5p[\lambda_m i_q + (L_d - L_q)i_d i_q] \quad (6)$$

$$T_e = T_L + D\omega_r + J\dot{\omega}_r \quad (7)$$

$D$  is the friction coefficient;  $J$  is the moment of inertia,  $\lambda_m$  is the flux linkage,  $T_e$  is the electromagnetic torque,  $T_L$  is the load torque.

After linearization of the above equations, the state-space model is obtained as follows:

$$A = \begin{bmatrix} -\frac{R}{L} & 0 & 0 \\ 0 & -\frac{R}{L} & -\frac{\lambda_m p}{L} \\ 0 & -\frac{3\lambda_m p}{2J} & -\frac{D}{J} \end{bmatrix}, \quad (8)$$

$$B_u = \begin{bmatrix} 0 \\ 1 \\ L \end{bmatrix}, B_w = \begin{bmatrix} 0 \\ 0 \\ -\frac{1}{J} \end{bmatrix},$$

$$C = [0 \quad 0 \quad 1]$$

where  $x = [i_d \quad i_q \quad \omega_r]^T$  is space-vector,  $u = [v_q]$  is input vector.

The state-space representation is written as

$$\begin{cases} \dot{x}(t) = Ax(t) + B_u u(t) + B_w w(t) \\ z(t) = C_z x(t) + D_{uz} u(t) + D_{wz} w(t) \end{cases} \quad (9)$$

where  $A \in \mathbb{R}^{n \times n}$ ,  $B_u \in \mathbb{R}^{m \times n}$ ,  $B_w \in \mathbb{R}^{n \times r}$ ,  $C_z \in \mathbb{R}^{p \times n}$

$D_{uz} \in \mathbb{R}^{p \times m}$ ,  $D_{wz} \in \mathbb{R}^{p \times r}$  and

$x = [i_d(t) \quad i_q(t) \quad \omega_r(t) \quad x_4(t)]^T$  is space vector,  $u = [v_q(t)]$  is input vector and  $z = [\omega_r(t)]$  is output vector.

The integrator is necessary to meet the reference tracking and constant disturbance rejection. Integral control also ensures the rejection of the constant disturbance (see Fig. 1).

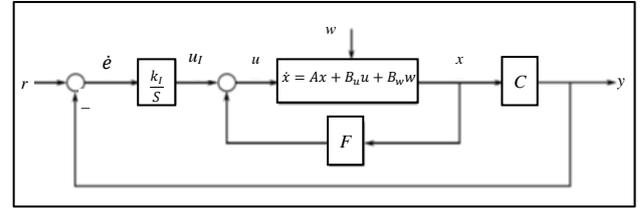


Figure 1: Integral control with full state feedback.

The plant model given in (9) is illustrated in Fig. 1. the system depicted in Fig. 1 can be written as

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix} + \begin{bmatrix} B_u \\ 0 \end{bmatrix} u + \begin{bmatrix} B_w \\ 0 \end{bmatrix} w + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r \quad (10)$$

$$y = [C \quad 0] \begin{bmatrix} x \\ e \end{bmatrix}$$

where  $r = \omega_{r\_ref}$  and  $u = Fx + K_i e$ .

The state variable  $x_4$  is the integral of the error signal obtained from the difference between the reference  $\omega_{r\_ref}$  and the output speed  $\omega_r$ . At the equilibrium state, the speed error is zero. Therefore,  $x_4$  is constant.

The input is considered to be a DC voltage. The disturbance vector  $w$  has been defined as a load torque.

Matrices of the state-space representation are as follows:

$$A = \begin{bmatrix} -\frac{R}{L} & 0 & 0 \\ 0 & -\frac{R}{L} & -\frac{\lambda_m p}{L} \\ 0 & -\frac{3\lambda_m p}{2J} & -\frac{D}{J} \\ 0 & 0 & K_i \end{bmatrix}, \quad (11)$$

$$B_u = \begin{bmatrix} 0 \\ 1 \\ L \\ 0 \\ 0 \end{bmatrix}, B_w = \begin{bmatrix} 0 \\ 0 \\ -\frac{1}{J} \\ 0 \end{bmatrix},$$

The remaining state-space matrices are:

$$C_z = [0 \ 0 \ 1 \ 0], \quad (12)$$

$$D_{uz} = [0], D_{wz} = [0]$$

### 3. PULSE WIDTH MODULATION INVERTER

Pulse width modulation is the process of modifying the width of the pulses in a pulse train, in which, the greater the control voltage, the wider the resulting pulses become. By using a sinusoid of the desired frequency as the control voltage for a PWM circuit, it is possible to produce a high-power waveform whose average voltage varies sinusoidally in a manner suitable for driving AC motors.

Fig. 2 shows circuit model of a single-phase inverter with a center-tapped grounded DC bus, and Fig. 3 illustrates principle of pulse width modulation.

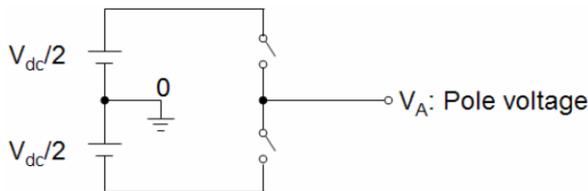


Figure 2: Circuit model of a single-phase inverter.

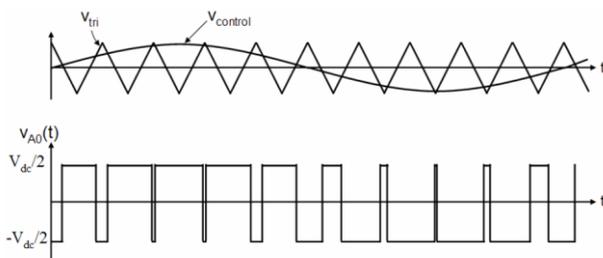


Figure 3: Pulse width modulation.

As depicted in Fig. 3, the inverter output voltage is determined as follows:

$$\begin{cases} V_{A0} = V_{dc}/2 & , V_{control} > V_{tri} \\ V_{A0} = -V_{dc}/2 & , V_{control} < V_{tri} \end{cases} \quad (13)$$

Also, the inverter output voltage has the following features:

- PWM frequency is the same as the frequency of  $V_{tri}$
- Amplitude is controlled by the peak value of  $V_{control}$
- Fundamental frequency is controlled by the frequency of  $V_{control}$

Modulation index (m) is defined as:

$$m = \frac{V_{control}}{V_{tri}} = \frac{\text{Peak of } (V_{A0})_1}{V_{dc}/2} \quad (14)$$

### 4. LINEAR PARAMETER VARIATION (LPV)

We consider that the stator resistance R and the friction coefficient D at the operating point are uncertain or time-varying parameters (Linear variation). It is worth to point out that the same procedure can be used to take into account more uncertain terms. Nevertheless, the more uncertainty is considered in the converter, the lower performance level can be assured. Thus, in order to deal with the changes in parameters D and R, we express the system matrices (9) as function of these parameters:

$$\begin{cases} \dot{x}(t) = A(\varphi)x(t) + B_u u(t) + B_w w(t) \\ z(t) = C_z x(t) + D_{uz} u(t) + D_{wz} w(t) \end{cases} \quad (15)$$

Only matrix A depends on the uncertain terms, which has been grouped in a vector  $\Phi$ . In a general case, the vector  $p$  consists of N uncertain parameters  $\varphi_1, \dots, \varphi_N$ , where each uncertain parameter  $\varphi_i$  is bounded between a minimum and a maximum value,  $\varphi_{i-min}$  and  $\varphi_{i-max}$ .

The admissible values of vector  $\Phi$  are constrained in a hyper rectangle in the parameter space  $\mathbb{R}^N$  with  $L = 2^N$  vertices  $\{v_1, \dots, v_L\}$ . The images of the matrix  $A(\varphi)$  for each vertex  $v_i$  corresponds to a set  $\{\mathcal{G}_1, \dots, \mathcal{G}_L\}$ . The components of the set  $\{\mathcal{G}_1, \dots, \mathcal{G}_L\}$  are the extrema of a convex polytope, noted  $Co\{\mathcal{G}_1, \dots, \mathcal{G}_L\}$ , which contains the images for all admissible values of  $\Phi$ , if the matrix  $A(\varphi)$  depends linearly on  $\Phi$ , that is:

$$A(\varphi) \in Co\{A_1, \dots, A_k\} := \left\{ \sum_{i=1}^k \alpha_i A_i : \alpha_i \geq 0, \sum_{i=1}^k \alpha_i = 1 \right\} \quad (16)$$

For an in-deep explanation of polytopic models of uncertainty see [10, 12, 13]. By using this parameter vector, we can bound the uncertainty inside a convex polytope.

Based on the uncertainty model described above, the synthesis objective is to find a state-feedback gain K ( $u = Kx$ ), where uncertainty is restricted to the following intervals:

$$\begin{aligned} R &\in [R_{min} \quad R_{max}] \\ D &\in [D_{min} \quad D_{max}] \end{aligned} \quad (17)$$

Note that the uncertain model is inside a polytopic domain formed by L=16 vertices for more accuracy. (see Fig. 4).

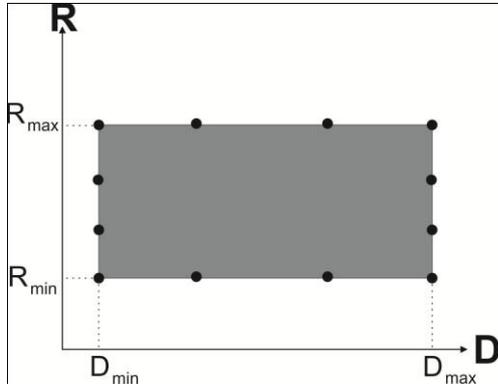


Figure 4: Variations of uncertain parameter's span.

The objectives of the design are to guarantee the stability, to assure a minimum level of perturbation rejection and to constraint the transient performance, for all the possible cases of  $A(\varphi)$ . In the next section, we will establish the LMI conditions which satisfy these objectives.

## 5. H INFINITY

This section introduces the concept of H infinity and presents the constraints used in the controller synthesis problem.

### A. Lyapunov-based stability

The use of matrix inequalities to demonstrate certain properties of dynamical systems have already been presented in about 1890, when Lyapunov has established his well-known stability method, whose linear case has been reproduced here for completeness [13].

Given a linear time-invariant (LTI) system

$$\dot{x} = Ax \quad (18)$$

The existence of a quadratic function of the form

$$V(x) = x^T Px > 0, \quad \forall x \neq 0 \quad (19)$$

that satisfies  $\dot{V}(x) < 0$  is a necessary and sufficient condition to assure that the system is stable (i.e. all trajectories converge to zero) [13]. Since  $V(x)$  has quadratic form, this condition is referred as quadratic stability. In this case, the condition  $\dot{V}(x) < 0$  can be rewritten as follows:

$$\begin{aligned} \dot{V}(x) = x^T (A^T P + PA)x > 0, \\ \forall x \neq 0 \end{aligned} \quad (20)$$

Thus, the system is stable if and only if there exists a symmetric matrix  $P$  that is positive definite (in the following, the notation  $P > 0$  means that the matrix  $P$  is positive definite) for which  $\dot{V}(x) < 0$ . Inequality (20) is satisfied if and only if the term  $(A^T P + PA)$  is negative definite, that is

$$\exists P > 0 \text{ s.t. } A^T P + PA < 0 \quad (21)$$

In this case,  $P$  is the matrix variable to be found to prove the stability. Ref. [14] has shown that these inequalities that presented linear dependence on the variables, which is called LMIs, hereafter, can be solved by convex optimization methods.

We will take advantage of such convex optimization methods that have been implemented in computer algorithms [10,11], in order to solve the LMIs that arise in the control of motor speed. In the following subsections, we introduce the LMIs to ensure the quadratic stability, to deal appropriately with disturbances and to constraint the pole placement of the closed-loop system.

### B. Quadratic stability LMIs

The following theorem [12] adapts the quadratic stability inequality (21) for a closed-loop system with a state feedback:

$$u = Kx \quad (22)$$

Theorem 1: System (9) is stabilizable by state feedback  $u = Kx$  if and only if there exists a symmetric matrix  $W \in \mathbb{R}^{n \times n}$  and a matrix  $Y \in \mathbb{R}^{m \times n}$  such that  $W > 0$ .

$$\begin{cases} W > 0 \\ AW + WA^T + B_u Y + Y^T B_u^T < 0 \end{cases} \quad (23)$$

A controller for such state feedback is given by  $YW^{-1}$ . The decomposition of  $K$  in matrix variables  $Y$  and  $W$  allows to satisfy the linearity condition of these inequalities. A detailed proof can be obtained in [12]. The case with polytopic uncertainty in matrix  $A$  directly extends by computing (23) at all the vertices  $\{\mathcal{G}_1, \dots, \mathcal{G}_L\}$  of the convex polytopic  $Co\{\mathcal{G}_1, \dots, \mathcal{G}_L\}$  [13]. This extension allows us to assure the quadratic stability of an uncertain plant for LPV control.

The  $H_\infty$  norm of a stable scalar transfer function  $f(s)$  is the peak value of  $|f(j\omega)|$  as a function of frequency [15]. It is used as a measure of the performance of a system, for example, to evaluate the minimum attenuation level of an external disturbance. Considering the transfer function  $H_\infty$  from disturbances  $w$  to outputs  $z$ , the  $H(s)$  norm of such system is equal to:

$$\|H(s)\|_\infty \triangleq \sup_{\omega(t) \neq 0} \frac{\|z\|_2}{\|w\|_2} \quad (24)$$

where  $\|\cdot\|_2$  and  $\|\cdot\|_\infty$  stand for the infinity and the

Euclidian norms, respectively.

The following theorem, adapted from [16], guarantees a maximum  $H_\infty$  norm  $\gamma$  (i.e. a minimum level of disturbance attenuation).

Theorem 2: System (9) is stabilizable by state-feedback  $u = Kx$  and  $\|z\|_2/\|w\|_2 < \gamma$  if and only if there exists a symmetric definite positive matrix  $W \in \mathbb{R}^{n \times n}$  and a matrix  $Y \in \mathbb{R}^{m \times n}$  such that the following inequality holds:

$$\begin{bmatrix} AW + WA^T + B_u Y + Y^T B_u^T & B_w & WC^T + Y^T D_u^T \\ B_w^T & -\gamma I & D_w \\ CW + D_u Y & D_w & -\gamma I \end{bmatrix} < 0 \tag{25}$$

A controller for such state feedback is given by  $K = YW^{-1}$ .

A proof is given in [16]. Note that satisfaction of inequality (25) implies satisfaction of (23) and therefore this theorem also ensures quadratic stability. The polytopic case is directly applicable by satisfying (25) for all the vertices  $\{\mathcal{G}_1, \dots, \mathcal{G}_L\}$  of the polytopic domain.

C. Pole placement LMIs

It is a desirable property of the closed-loop system that its poles are located in a certain region of the complex plane, in order to assure some dynamical properties like overshoot and settling time. In [17], a region of the complex plane  $S(\alpha, \rho, \theta)$ , depicted in Fig. 2, is defined such that the (complex) poles of the system in the form of  $x \pm jy$  satisfy

$$x < -\alpha < 0, \quad |x \pm jy| < \rho,$$

$$y < \cot(\theta) x$$

In such a case, the poles of the system,  $x \pm jy = -\zeta\omega_n \pm j\omega_d$  ensure a certain damping at the desired rate [18]. The presented region is equivalent to a minimum decay rate  $\alpha$ , a minimum damping ratio  $\zeta > \sin \theta$  and a maximum damped natural frequency  $\omega_d < \rho \cos \theta$ , where  $\omega_d = \omega_n \sqrt{1 - \zeta^2}$ .

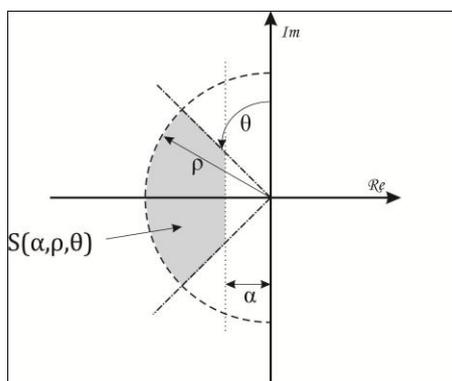


Figure 5: Pole Placement region.

The following theorem allows to constraint the location of the closed-loop poles in the region  $S(\alpha, \rho, \theta)$ .

Theorem 3: The closed-loop poles of the system (1) with a state-feedback  $u = Kx$  are inside the region  $S(\alpha, \rho, \theta)$ , If there exists a symmetric definite positive matrix  $W$  and a matrix  $Y$  such that

$$AW + WA^T + B_u Y + Y^T B_u^T + 2\alpha W < 0 \tag{25}$$

$$\begin{bmatrix} -\rho W & WA^T + Y^T B_u^T \\ AW + B_u Y & -\rho W \end{bmatrix} < 0 \tag{26}$$

$$\begin{bmatrix} \cos \theta (AW + WA^T + B_u Y + Y^T B_u^T) \\ \sin \theta (-AW + WA^T - B_u Y + Y^T B_u^T) \\ \sin \theta (AW - WA^T + B_u Y - Y^T B_u^T) \\ \cos \theta (AW + WA^T + B_u Y + Y^T B_u^T) \end{bmatrix} < 0 \tag{27}$$

and  $K = YW^{-1}$  is the state feedback gain.

A detailed proof is reported in [17], where the pole placement in generic regions of the complex plane is explored. A different point of view on robust pole placement is given in [19]. Once again, the polytopic case directly extends by satisfying (27)–(29) for all  $\{\mathcal{G}_1, \dots, \mathcal{G}_L\}$ .

Therefore, by taking into account these conditions in each vertex of the convex polytope  $Co\{\mathcal{G}_1, \dots, \mathcal{G}_L\}$ , we will ensure a proper performance despite uncertainty.

We summarize the LMI synthesis method as follows. The design of a robust control for a boost converter consists of solving inequalities (25) and (27)–(29) to find the matrix variables  $Y$  and  $W$  that minimize the  $H_\infty$  norm  $\gamma$  and that satisfy the pole placement constraints of the region  $S(\alpha, \rho, \theta)$  for all the extreme of the polytopic model  $\{\mathcal{G}_1, \dots, \mathcal{G}_L\}$  Under condition:

$$\min_{Y, W} \gamma \quad (25), (27), (28) \text{ and } (29) \tag{28}$$

$$\forall \{\mathcal{G}_i\}, \quad i = 1, \dots, L$$

It is necessary to remark that the stability and the  $H_\infty$  bound of the closed-loop system is guaranteed for arbitrarily fast changes in  $D$  and  $R$ . However, the pole placement constraint is only satisfied if the time-dependent parameters changes are slowly enough to recover the steady state of the system.

6. SIMULATION RESULTS

In this section, we present the results of the LPV design method proposed above. First, we identify the parameter values of the PMSM motor. Next, the state-feedback gain  $K$  is numerically calculated. The resulting controller has been simulated with a switched model of the PWM inverter. We have compared the simulated response of the proposed controller with a conventional PI controller, in order to evaluate the robustness and performance of our approach.

As it has been stated before, the synthesis objective is to minimize  $\gamma$  while a pole placement constraint  $S(\alpha, \rho, \theta)$  is satisfied. In order to have the poles of the system inside the valid frequency range of the model,  $\rho$  has been set to 1/10 of the switching frequency. For a minimum damping ratio of 0.36,  $u$  has been set to 1098. Finally, for a fast decay rate, we have tested several different values of  $a$ . For the present case, the parameter  $\alpha$  has been set to 10.98; a higher value of  $a$  results in an unfeasible problem for this parameter set.

Solving problem (30) using YALMIP Toolbox in MATLAB software [10], yields a state-feedback controller  $K=[0.0014 \ -3.8807 \ -41.9004]$  and a guaranteed  $H_\infty$  bound from disturbances to outputs of  $\gamma=0.8$ .

Simulation block diagram is shown in Fig. 6. In this figure,  $H_\infty$  controller, tracking integrator, PWM inverter, PMSM and sensor blocks are connected together.

Motor's parameters used in simulations are given in Table 1:

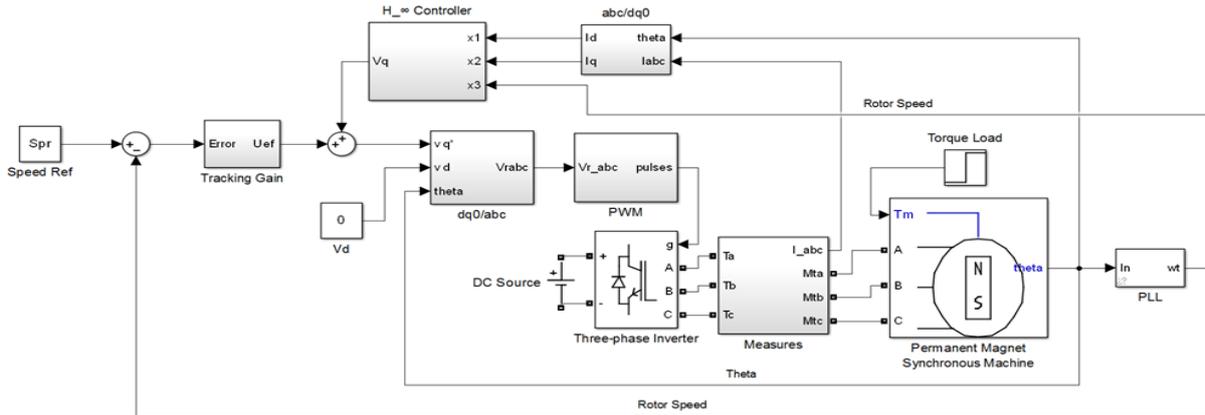


Figure 6: Simulation block diagram ( $H_\infty$ -controller, tracking integrator, PWM Inverter, PMSM and sensor blocks is connected together).

TABLE 1  
PMSM MOTOR PARAMETERS

row	Parameter	value
1	Machine power	1KW
2	Rated current	6.5A
3	Pole pair number (p)	2
4	d-axis inductance $L_d$	4.5mH
5	q-axis inductance $L_q$	4mH
6	Stator resistance R	0.56 $\Omega$
7	Machine inertia J	$2.08 \times 10^{-3} \text{Kg.m}^2$
8	Friction coefficient D	$3.9 \times 10^{-3} \text{Nm.s.rad}^{-1}$

First, we have obtained open loop response under the influence of uncertainties and external disturbances. We have changed the friction coefficient D and stator resistance R by a variation of +150% from the nominal values at  $t=1.5, 2.5$  Sec., respectively, and force load torque to 3 N.m. at  $t=4$  Sec. The result obtained by the above changes, is shown in Fig. 7.

After connection of the controller and inverter to the motor, repeat simulations, we obtained results via improved performance in face of parameter variation and external disturbances (See Fig. 8).

The Motor current and torque are shown in Fig. 9. In this figure, the speed change with parameter variations is good.

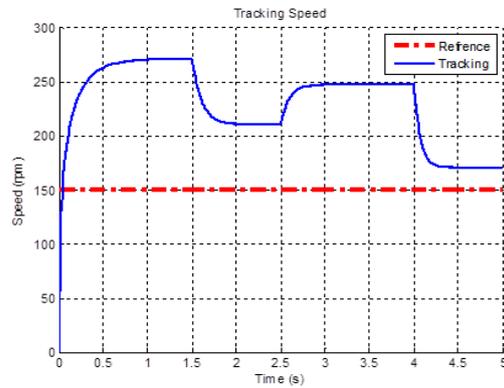


Figure 7: Open loop response under the influence of uncertainties and external disturbances.

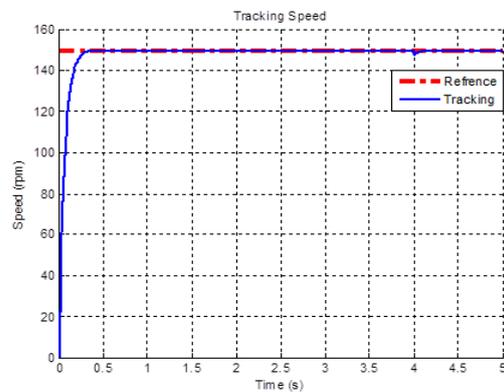


Figure 8: Step response and speed tracking under the influence of uncertainties and external disturbances with  $LPV H_\infty$  controller.

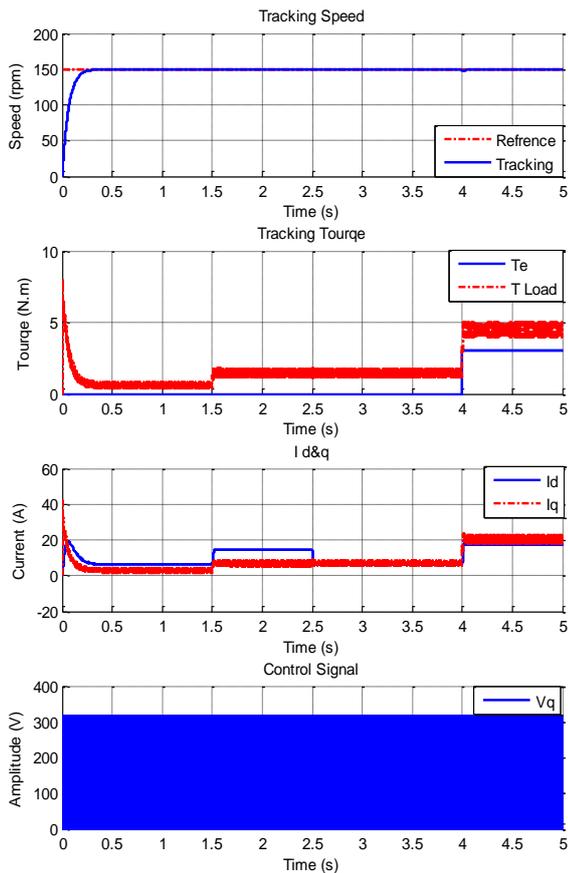


Figure 9: Speed, torque, current and voltage in speed tracking under the influence of uncertainties and external disturbances with LPV  $H_\infty$  controller.

The output signal from inverter is shown in Fig. 10.

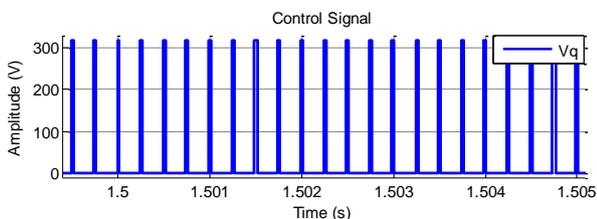


Figure 10: Inverter output signal in speed tracking under the influence of uncertainties and external disturbances with LPV  $H_\infty$  controller.

The next step is comparison with PI controller. We have used a PI controller with  $K_i = 100$  and  $K_p = 2$ . The results obtained are shown in Fig. 11.

It can be seen, PI controller has a step response in  $t_{settling} = 0.38 \text{ Sec}$  with 21.4% overshoot, but LPV  $H_\infty$  controller has a response in  $t_{settling} = 0.54 \text{ Sec}$  with 0% overshoot.

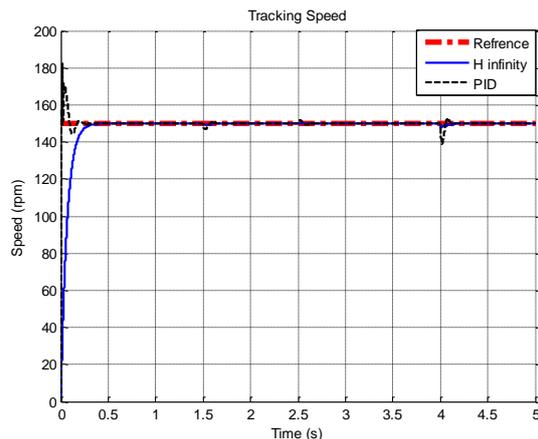


Figure 11: Tracking speed under the influence of uncertainties and external disturbances with LPV  $H_\infty$  controller.

The comparison of other results are shown in Fig. 12 to 14.

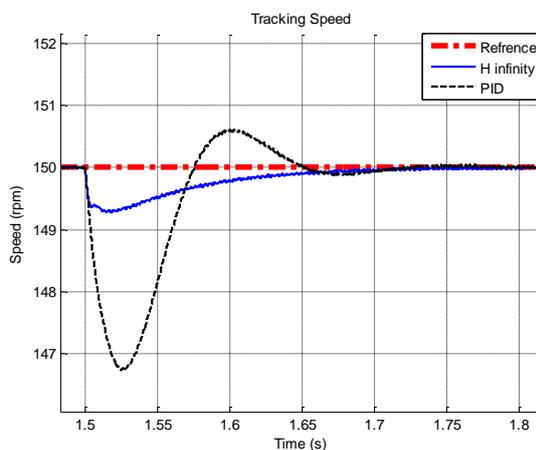


Figure 12: Tracking speed under the variation of friction coefficient D with LPV  $H_\infty$  & PI controllers.

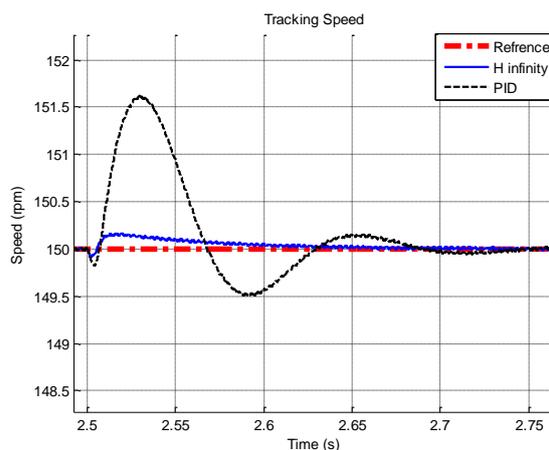


Figure 13: Tracking Speed under the variation of Stator resistance R with LPV  $H_\infty$  & PI controllers.

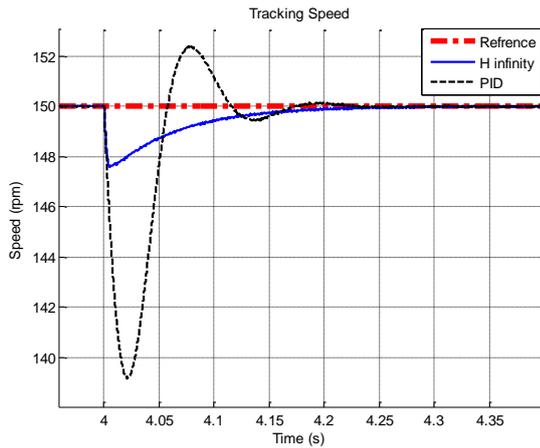


Figure 14: Tracking speed under the variation of load torque with  $LPV H_\infty$  & PI controllers.

The comparison of the results of the control parameters obtained by PI and  $H_\infty$  controllers are shown in table 2.

TABLE 2  
CONTROL PARAMETERS COMPARISON,  $LPV H_\infty$  CONTROLLER WITH PI CONTROLLER

row	Variation Type	Control Parameter	PI Controller	$LPV H_\infty$ Controller
1	Friction Coefficient D	Rise time (Sec)	0.075	0.15
2		Settling time (Sec)	0.20	0.15
3		Under Shoot (%)	2.16	0.48
4	Stator Resistance R	Rise time (Sec)	0.068	0.175
5		Settling time (Sec)	0.28	0.175
6		Over Shoot (%)	1.06	0.1
7	Load Torque TL	Rise time (Sec)	0.057	0.38
8		Settling time (Sec)	0.31	0.38
9		Under Shoot (%)	7.18	1.54

7. CONCLUSION

This paper developed a methodology to design a robust  $LPV H_\infty$  controller in order to ensure good performance robustness under uncertainty perturbations. An efficient algorithm based on iterative technique was proposed to minimize the  $H_\infty$  norm of an augmented plant, including nominal model of the PMSM and the controller K.

The controller has been implemented with success on a PMSM. The results of the PI controller and the proposed method have been compared in Table 2, confirming the good performance of the latter.

8. APPENDIX

a. abc to dq0 Conversion equation:

$$\begin{bmatrix} V_q \\ V_d \\ V_0 \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos \theta_r & \cos (\theta_r - 120) & \cos (\theta_r + 120) \\ \sin \theta_r & \sin (\theta_r - 120) & \sin (\theta_r + 120) \\ 1/2 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

b. dq0 to abc Conversion equation:

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \frac{3}{2} \begin{bmatrix} \cos \theta_r & \sin \theta_r & 1 \\ \cos (\theta_r - 120) & \sin (\theta_r - 120) & 1 \\ \cos (\theta_r + 120) & \sin (\theta_r + 120) & 1 \end{bmatrix} \begin{bmatrix} V_q \\ V_d \\ V_0 \end{bmatrix}$$

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